KTH Mathematics

Some problems to be solved by using GAMS

1. A transportation problem

In this exercise we are to study a model for transportation planning. We will improve our model during the way from the simplest transportation planning model to a small model for transportation planning in connection to e.g. mobilization.

The aim of the exercise is to give You an insight in modulation development and also to give some practical training in the modulation language GAMS. You are not supposed to learn GAMS in detail yet, but are supposed to copy the syntax from known examples. Begin by collecting the files `trans1.gms`, `trans2.gms`, `trans3.gms`, and `trans4.gms` from the course web-page.

Now we will start from the following simple transportation problem.

We wish to supply Stockholm and Boden with goods from Sollefteå and Gävle using the least amount of money possible. The demand in Stockholm is 40 ton and in Boden 10 ton. In Sollefteå and in Gävle there are 30 ton of goods. The transportation cost is proportional to the distance between the cities.

1.1 Study the file `trans1.gms` with the editor and make sure that you understand

- the different parts `SETS, TABLE, PARAMETER, SCALAR, VARIABLES, MODEL, EQUATIONS, SOLVE` and `DISPLAY`,
- the equations (objective function and constraints),
- the modell.

1.2 Run the file `trans1.gms` by the command: `gams trans1`.

1.3 Study the result file `trans1.lst` with the editor. Especially the result is in the end of the file. What is the optimal way of performing the transportation? What is the totalcost for the optimal way of performing the transportation?

1.4 We will now extend the model with more goods. We imagine a demand of tanks and of terrain-cars in Stockholm and in Boden, and that these are available in Sollefteå and in Gävle. The transportation capacity between the cities are limited. The file `trans2.gms` contain all necessary data you need for the model. Variables and equations needed are defined and declared. However, one set of declared equations are missing, the one limiting the transportation capacity, `CAPCON(I,J)`. Declare the equations `CAPCON(I,J)` and add them to the already existing equations in `trans2.gms`. (OBS Save the changes).

1.5 Run the file `trans2.gms`. If you obtain message of error summon the teacher for immediate assistance. Study the out-data file `trans2.lst`. What is the optimal way of performing the transportation and how much does it cost?

1.6 Now we will expand our model once more. This time we introduce different ways of performing the transportation, train and by airplane. From the depot we have limited supply of transportation capacity by each means of transport. Necessary data are found in the file `trans3.gms`. Formulate the optimization problem corresponding to this data. Complement the file `trans3.gms` with variables and definitions of equations. Run and analyze the file `trans3.lst`.

1.7 Now we imagine that the needs in Stockholm and Boden vary in time, over the following week, and we will fulfill these from the given storage facilities in Sollefteå and Gävle. The transportation time by train is two days and by airplane is one day irrespectively of between which two cities the transportation takes place. Necessary data are found in the file `trans4.gms`. Formulate the optimization problem corresponding to this data. Comlement the file `trans4.gms` with variables and definitions of equations (You have been helped with the equations `SATDEM(J,V,T)`). Run and analyze the results.

2. The diet problem

Dietists on a hospital wish to develop a computerized menu planning system. As a start, one wants to compose a lunch menu. The menu is divided into three categories: vegetables, meat and dessert. At least one component from each category has to be included in the menu. The cost per portion of some of the suggested components and their nutritional content in terms of carbohydrates, vitamins, proteins and fats are listed in the table below.

	Carbohydrates	Vitamins	Proteins	Fats	The cost ($)
<u>Vegetables</u>					
Peas	1	3	1	0	0.10
Beans	1	5	2	0	0.12
Okra	1	5	1	0	0.13
Corns	2	6	1	2	0.09
Pasta	4	2	1	1	0.10
Rice	5	1	1	1	0.07
<u>Meat</u>					
Chicken	2	1	3	1	0.70
Beef	3	8	5	2	1.20
Fish	3	6	6	1	0.63
<u>Dessert</u>					
Orange	1	3	1	0	0.28
Apple	1	2	0	0	0.42
Pudding	1	0	0	0	0.15
Jello	1	0	0	0	0.12

Assume that the demand of nutritional content per meal is at least 5, 10, 10 and 2 units of carbohydrates, vitamins, proteins and fats.

2.1 Formulate the menu planning problem as an integer programming problem (integer with 0-1 variables) and use `GAMS` to solve it. There is a start in the file `menu.gms`. You only need to complement it by some necessary equations and some more.

2.2 Modify the `GAMS` program such that it suggest a second menu, which does not contain any of the components in the first menu. Hint: Use the internal database in `GAMS`.

2.3 Modify the model such that it suggests a menu that have the lowest fat content as possible, but does not cost more than $1.20.

3. Car rental

A car rental company in New York has rental offices at the three major airports Kennedy, La Guardia and Newark in addition to two Manhattan offices, the north Manhattan office and the south Manhattan office. Sunday nights most rental cars are returned at the Manhattan offices when people return from weekend trips. Monday morning most cars are needed at the airports, and hence the cars must be transported from Manhattan to the airports.

The north and the south Manhattan offices have 180 respective 100 cars available. At Kennedy, La Guardia and Newark there are 20, 30 and 25 cars available, respectively. The transportation cost from the Manhattan offices to the airports is given in dollar as

	Kennedy	La Guardia	Newark
Manhattan south	6	10	12
Manhattan north	8	8	6

The demand in the morning at the airport is uncertain, but from previous experience it is known that the demand is approximately exponentially distributed, i.e., the demand has density

$$\phi(r) = \frac{1}{m} e^{-r/m},$$

where m is the mean. The mean demand at Kennedy Airport is 150, at La Guardia 100 and at Newark 80 cars.

If the demand at an airport exceeds the supply the average loss per car is estimated to 22 dollars.

3.1 Formulate an optimization problem for minimizing the total transportation cost plus expected loss due shortage of cars. Model in `GAMS` and solve.

Hint: $\int_y^\infty (r - y)\frac{1}{m} e^{-r/m} dr = m e^{-y/m}$.

3.2 Assume that one also is interested in investigating if supply exceeds demand, due to shortage of parking space at the airports. If there are more than 30 cars left on the Monday, they must be parked at a remote parking area to a cost of 2, 2, and 3 dollars per car, at the respective airport. Modify your model to take this into account.

Hint: $\int_0^y (y-r)\frac{1}{m}e^{-r/m}dr = y + me^{-y/m} - m.$