KTH Matematik

Exam in SF2862 Stochastic Decision Support Models.
Friday March 13, 2009, 8.00–13.00

Examiner: Krister Svanberg, tel. 790 71 37

Allowed tools: Pen/pencil, ruler and eraser. A formula-sheet is handed out.
No calculator is allowed!

Solution methods: Unless otherwise stated in the text, the problems should be solved by systematic methods, which do not become unrealistic for large problems. Conclusions should be motivated. If you use methods other than what has been taught in the course, you must explain very carefully.

Note! Personal number must be written on the title page. Write only one excercise per sheet. Number the pages and write your name on each sheet!
25 points, including bonus points from the homeworks, are sufficent for passing the exam. For 23-24 points, a completion to a passing grade may be made within three weeks from the date when the results of the exam have been reported. Contact the examiner as soon as possible for such a completion.

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1. The airline Happy has noticed that when a flight is fully booked then typically some passenger do not show up at the departure time, so called "no-shows". This gives the possibilty for Happy to do overbooking, i.e. to book more passengers on a flight than there are seats. Based on long-time experiences, Happy claims that for a certain type of flights (which will be considered here) the number of no-shows is a discrete random variable ξ which has (approximately) the following distribution:

$P(\xi = j) = \dfrac{5 - |\,j - 5\,|}{25}$ for $j = 1, \ldots, 9$, while $P(\xi = j) = 0$ for all other j.

They also claim the following: Each empty seat on the considered fligth causes Happy a cost of 300 Euro, compared to if one more passenger had been booked. Each "bumped" passenger (= a passenger who has been booked and show up at the departure time but does not get a seat) causes Happy a cost of 600 Euro, compared to if one less passenger had been booked, since the bumped passenger must be generously compensated.

Happy wants to minimize the sum of the expected values of the above costs.
Help them to calculate the optimal number of overbookings. . . . . (9p)

If you don't manage to deal with the above discrete random variable, you may instead do the approximation that ξ is a continuous random variable with density function $f_\xi(x) = \dfrac{5 - |\,x - 5\,|}{25}$ for $x \in [0,\, 10]$, while $f_\xi(x) = 0$ for all other x, and round the optimal solution to the nearest integer. But then you will get **at most 7p!**

2. A queueing system consist of three different service facilities, F1, F2 and F3. Customers arrive to the system according to a poisson process with rate 20 customers/hour. Each arriving customer first go to facility F1, where there are two parallel servers, each with an exponentially distributed service time with mean 3 minutes/customer.
A customer who leaves facility F1 goes to facility F2 with probability 0.4, to facility F3 with probability 0.3, and leaves the whole system with probability 0.3.
In each of the facilities F2 and F3 there is a single server with exponentially distributed service time with mean 2 minutes/customer (in both F2 and F3).
A customer who leaves facility F2 goes to facility F3 with probabilty 3/4 and leaves the whole system with probability 1/4. A customer who leaves facility F3 goes to facility F2 with probabilty 2/3 and leaves the system with probability 1/3.
Assume that the system is in steady state.

(a) What is the average number of customers in the system? ....(4p)

(b) What is the probability that there is exactly one customer in the system (at a randomly chosen point in time)? ....(4p)

(c) What is the expected remaining time in the system for a customer who has just arrived to facility F2? ....(4p)

3. A certain repair shop receive broken engines from aircrafts. Each arriving engine has an error either of type A or of type B (not both), but it is not known which. What is known is that the probabilty of an error of type A is 0.6 and the probabilty of an error of type B is 0.4.

There are two types of repairs, RA and RB. RA takes 40 hours to complete and it will cure an error of type A, but not an error of type B. RB takes 30 hours to complete and it will cure an error of type B, but not an error of type A. This means that if one starts with the "wrong" repair, the total time for repairing the engine will be 70 hours (30+40 or 40+30).

Before deciding which repair to start with, a test can be done. Unfortunately, this test does not tell with certainty what type of error there is. If the engine has an error of type A, then the test will tell the correct result (A) with probability 2/3 but the wrong result (B) with probability 1/3. If the engine has an error of type B, then the test will tell the correct result (B) with probability 3/4 but the wrong result (A) with probability 1/4.

Assume that the time required for doing the test is T hours, and that the repair shop wants to minimize the expected total time for curing the engine. For which values on T is it optimal to do the test before starting the repair? ....(8p)

4. Consider a certain infinite-horizon discrete-time system. By the end of each time period, the system is in one of two states, S or T, and there are two different actions to choose between, A or B.

If action A is chosen, the probability that the state will change during the next period is 0.25, while if action B is chosen, the probability that the state will change during the next period is 0.5.

The cost during the next period depends on the current state, the choice of action, and the state at the end of the next period. More precisely, the following hold:

If the current state is S, action A is chosen, and the system remains in state S, then the cost during the next period is 24 units. If the current state is S, action A is chosen, and the system changes to state T, then the cost during the next period is 28 units. We write this as $C(S{-}A{-}S) = 24$ and $C(S{-}A{-}T) = 28$.

With the same notations, the following hold:

$C(S{-}B{-}S) = 24$ and $C(S{-}B{-}T) = 40$.

$C(T{-}A{-}T) = 6$ and $C(T{-}A{-}S) = 14$.

$C(T{-}B{-}T) = 0$ and $C(T{-}B{-}S) = 2$.

(a) Assume that the expected average long-term cost should be minimized (without any discounting). Verify that an optimal policy is to choose action B if the system is in state S, and action A if the system is in state T. . . . . (4p)

(b) Formulate the corresponding LP problem, and use (a) to calculate an optimal solution of this LP problem. . . . . (4p)

(c) Now assume that a discount factor $\alpha = 0.8$ should be used, and that the objective is to minimize the expected total discounted cost. Verify that the policy suggested in (a) is no longer optimal. (It is not required that you calculate an optimal policy.) . . . . (4p)

5. On the round table in front of Captain Hook, there are 7 seemingly identical coins. He happens to know that exactly one of them is false, and he wants to find out which one. The false coin has lower weight than the others, and to Hook's disposal is a balance with two bowls. If he puts k coins in each bowl, the balance will tell him if the two bowls contain equal weight or if one of the bowls contains lower weight than the other, and in that case which one.

Hook has a reputation of beeing efficient, so he wants to use a strategy which minimizes the *expected* number of balance trials.

Let $V(n)$ be the *expected* number of balance trials (weighings) needed to identify the false coin when it is known to be among n specific coins, and an optimal strategy is used. In particular $V(1) = 0$.

(a) Formulate a recursive equation for $V(n)$. . . . . (3p)

(b) Use this recursive equation to calculate $V(n)$ for $n = 2, 3, 4, 5, 6$ and 7. . . (4p)

(c) Explain the optimal strategy for Captain Hook. . . . . (2p)

Good luck!