# GENERALIZATION OF FUNK–HECKE THEOREM TO THE HYPERBOLIC SPACES CASE

TATYANA V. SHTEPINA

Ukraine

The Funk–Hecke theorem asserts that surface spherical harmonics are eigenvectors for the broad class of integral operators (integration over the sphere surface) whose kernels depend on the angle $\rho$ between vectors $\xi$ and $\eta$ only ($|\xi| = |\eta| = 1$). In particular, for important from physical point of view kernel of the form $\exp(i\alpha\rho)$ the eigenvalue is, according to Hecke, $2\pi i^n\sqrt{\frac{2\pi}{\alpha}}J_{n+1/2}(\alpha)$, where $n$ is degree of spherical harmonic and $J_k(\alpha)$ is Bessel function.

Let $\mathbb{R}^{n-1,1}$ be the pseudoeuclidean space endowed with bilinear form

$$[x, y] = -x_1y_1 - \dots - x_{n-1}y_{n-1} + x_ny_n.$$

We denote by $L^2(S_H, d\mu)$ the space of square–integrable functions defined on hyperboloid $S_H = \{x|[x, x] = 1, x_n > 0\}$, $d\mu$ is measure on $S_H$, bilaterally invariant with respect to $SO_0(n-1, 1)$. There is an orthogonal basis in $L^2(S_H, d\mu)$ consisting from functions $H^{n\sigma}_K$ [1,ch X] known as hyperbolic harmonics of homogeneity degree $\sigma$.

**Theorem (generalization of Funk–Hecke one).** *Let $F(x)$ be a function of real variable such that*

(1) $F(x) \in L^1(-\infty, \infty) \cap L^2(-\infty, \infty)$

(2) $F(x) = \begin{cases} O\left(e^{-\mu x}\right) & \text{when } x \to +\infty \\ O\left(e^{\lambda x}\right) & \text{when } x \to -\infty \quad (\lambda, \mu > 0) \end{cases}$

(3) $F(x)$ *allows an analytic continuation to the function $F(x)$ of complex variable which is bounded and analytical on the lower half–plane.*

*Let $H^{n\sigma}_K(\xi)$ be an arbitrary hyperbolic harmonic of homogeneity degree $\sigma$. Then for any vector $\eta$ with $[\eta, \eta] = 1$ the next equality holds:*

$$\int\limits_{[\xi,\xi]=1} F([\xi, \eta])H^{n\sigma}_K(\xi)\, d\mu = \lambda^n_\sigma H^{n\sigma}_K(\eta)$$

## REFERENCES

1. *Vilenkin N.Ja., Special functions and theory of group representation*, Moskva: Nauka, 1991.

*E-mail address*: shtepina@stels.net

1991 *Mathematics Subject Classification.* 45P05.

*Key words and phrases.* hyperbolic harmonic, intertwining operator, quasiregular representation, Paley–Wiener theorem.

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX