RATES OF $A-$STATISTICAL CONVERGENCE OF POSITIVE LINEAR OPERATORS

O. DUMAN AND C. ORHAN

ABSTRACT. In the classical summability setting rates of summation have been introduced in several ways (see, e.g., [10], [21], [22]). The concept of statistical rates of convergence, for nonvanishing two null sequences, is studied in [13]. Unfortunately no single definition seems to have become the “standard” for the comparison of rates of summability transforms. The situation becomes even more uncharted when one considers rates of $A-$statistical convergence. For this reason various ways of defining rates of convergence in the $A-$statistical sense are introduced in [6].

In the present paper, using the concepts of [6], we study rates of $A-$statistical convergence of sequences of positive linear operators mapping the weighted space C_{ρ_1} into the weighted space B_{ρ_2} where ρ_1 and ρ_2 are weight functions satisfying the condition

$$\lim_{|x|\to\infty} \frac{\rho_1(x)}{\rho_2(x)} = 0;$$

and

$$B_{\rho_2} := \{f : f : \mathbb{R} \to \mathbb{R},\ |f(x)| \leq M_f\, \rho_2(x) \text{ for all } x \in \mathbb{R}\}$$

and

$$C_{\rho_1} := \{f \in B_{\rho_1} : f \text{ is continuous on } \mathbb{R}\}$$

(here M_f is a constant depending on f).

Note that the classical Korovkin type approximation theory may be found in [1], [4], [20] while its further extensions studied via $A-$statistical convergence may be viewed in [6], [7], [15].

Recall that the sequence (x_n) is said to be $A-$statistically convergent to L if, for every $\varepsilon > 0$;

$$\lim_j \sum_{n:|x_n - L| \geq \varepsilon} a_{jn} = 0$$

where $A = (a_{jn})$ is a non-negative regular matrix (see, e.g., [2], [3], [9], [23]. The case in which $A = C_1$; the Cesáro matrix, $A-$statistical convergence reduces to statistical convergence [8], [11], [12].

1991 Mathematics Subject Classification. Primary 41A25, 41A36, 47B38 ; Secondary 40A05.

Key words and phrases. $A-$ density, $A-$statistical convergence, sequence of positive linear operators, weight function, weighted space, modulus of continuity, the Korovkin theorem.

Section Number. 10 (Functional Analysis).

References

[1] F. Altomare and M. Campiti, Korovkin type Approximation Theory and its Application, Walter de Gruyter Studies in Math. 17, de Gruyter&Co., Berlin, 1994.
[2] J. S. Connor, On strong matrix summability with respect to a modulus and statistical convergence, Canad. Math. Bull. 32 (1989), 194-198.
[3] J. S. Connor and J. Kline, On statistical limit points and the consistency of statistical convergence, J. Math. Anal. Appl. 197 (1996), 392-399.
[4] R. A. Devore, The Approximation of Continuous Functions by Positive Linear Operators, Lecture Notes in Math. 293, Springer-Verlag, Berlin, 1972.
[5] O. Doğru, Weighted approximation of continuous functions on the all positive axis by modified linear positive operators, Int. J. Comput. Numer. Anal. Appl. 1 (2002), 135-147.
[6] O. Duman, M. K. Khan and C. Orhan, A ¡ Statistical convergence of approximating operators, Math. Inequal. Appl. 6 (2003), 689-699.
[7] O. Duman and C. Orhan, Statistical approximation by positive linear operators, Studia Math. 161 (2004), 187-197.
[8] H. Fast, Sur la convergence statistique, Colloq. Math. 2 (1951), 241-244.
[9] A. R. Freedman and J. J. Sember, Densities and summability, Pacific J. Math. 95 (1981), 293-305.
[10] J. A. Fridy, Minimal rates of summability, Canad. J. Math. 30 (1978), 808-816.
[11] J. A. Fridy, On statistical convergence, Analysis 5 (1985), 301-313.
[12] J. A. Fridy and C. Orhan, Statistical limit superior and limit inferior, Proc. Amer. Math. Soc. 125 (1997), 3625-3631.
[13] J. A. Fridy, H. I. Miller and C. Orhan, Statistical rates of convergence, Acta Sci. Math. (Szeged) 69 (2003), 147-157.
[14] A. D. Gadjiev, Theorems of the type of P. P. Korovkin's theorems, Mat. Zametki 20 (1976), 781-786 (in Russian).
[15] A. D. Gadjiev and C. Orhan, Some approximation theorems via statistical convergence, Rocky Mountain J. Math. 32 (2002), 129-138.
[16] A. D. Gadjiev, The convergence problem for a sequence of positive linear operators on unbounded sets, and theorems analogous to that of P. P. Korovkin, Soviet Math. Dokl. 15 (1974), 1433-1436.
[17] G. H. Hardy, Divergent Series, Oxford Univ. Press, London, 1949.
[18] E. Kolk, The statistical convergence in Banach spaces, Acta Et Commentationes Tartuensis 928 (1991), 41-52.
[19] E. Kolk, Matrix summability of statistically convergent sequences, Analysis 13 (1993), 77-83.
[20] P. P. Korovkin, Linear operators and Theory of Approximation, Hindustan Publ. Co., Delhi, 1960.
[21] M. Marouf, Summability Matrices that Preserve Various Types of Sequential Equivalence, Ph. D. Dissertation, Department of Mathematics, Kent State University, Kent, Ohio, 1989.
[22] H. I. Miller, Rates of convergence and topics in summability theory, Radovi Akademije Nauka i Umjetnosti BiH, LXXIV / 22 (1983), 39-55.
[23] H. I. Miller, A measure theoretical subsequence characterization of statistical convergence, Trans. Amer. Math. Soc. 347 (1995), 1811-1819.

O. Duman
Department of Mathematics, Faculty of Science, Ankara University,
Tandoğan 06100, Ankara, TURKEY.
E-mail Address: oduman@science.ankara.edu.tr

C. Orhan
Department of Mathematics, Faculty of Science, Ankara University,
Tandoğan 06100, Ankara, TURKEY.
E-mail Address: orhan@science.ankara.edu.tr