Stationary Solutions to 3D Navier–Stokes Equations with Random Force.

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Abstract

We consider 3D Navier–Stokes system in the Fourier space with regular forcing given by a stationary in time stochastic process satisfying a smallness condition. We explicitly build stationary solution to the system, study its properties and prove the uniqueness theorem for this solution. Moreover we prove the following "One Force — One Solution" principle: the unique stationary solution at time t is presented as a functional of the realization of the forcing in the past up to t. The explicit construction of the solution is based upon the stochastic cascade representation introduced in [1] and developed in [2].

Keywords: 3D Navier–Stokes system, random force, stationary solutions, stochastic cascades.

References

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