We give some results on the path properties of the solution of the stochastic partial differential equation

$$(\partial_{tt}^2 - \Delta_3)u(t,x) = \sigma(u(t,x))F(t,x) + b(u(t,x)), \tag{1}$$

with initial conditions $u(0,x) = v_0(x)$, $\partial_t u(0,x) = \tilde{v}_0(x)$, where $(t,x) \in [0,T] \times R^3$ and Δ_3 denotes the Laplacian on R^3.

The process $F(t,x)$ is Gaussian, white in time and correlated in space; the correlation measure is of the form $\Gamma(dx) = \varphi(x)f(x)dx$, where φ is smooth and $f(x) = |x|^{-\beta}$, with $\beta \in (0,2)$. Hence, Γ is singular and could include small long-term correlations.

A rigourous formulation of equation (1) is given be means of a stochastic integral for non stationary processes allowing distribution-valued integrands ([2]).

We prove that, uniformly with respect to $t \in [0,T]$, the solution of (1) takes its values in some fractional Sobolev space, a.s. Therefore, the Sobolev imbeddings imply α-Hölder continuity in the x variable. Then, we prove γ-Hölder continuity in t, a.s., uniformly with respect to x on bounded sets. This yields joint Hölder continuity in (t,x). The values of α and γ depend on the regularity of v_0, $\tilde{v}_0$ and the characteristics of the noise.

With techniques of Gaussian stationary processes ([1]), our results are proved to be optimal.

The proof relies on a careful analysis of the $L^p(\Omega)$-moments of increments of the stochastic integrals, using a spectral approach. Sharp results are obtained with fractional mean value arguments based on properties of Riesz potentials.

In [3], similar results for the wave equation in dimension two have been established. The methods of the proofs cannot been used in our setting. In fact, the more degenerate character of the equation in dimension three requires more sophisticated tools.

The results are joint work with R. Dalang, EPFL, Switzerland.

References

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