An explanation of G. Galilei's paradox [1], p. 140-146, can be obtained by means of some conditions, which make it possible to divide all injective mappings $\varphi: \ N \rightarrow N$ into five classes: finitely surjective, potentially surjective, potentially antisurjective, finitely antisurjective and total antisurjective mappings. In particular, the following statements are proved:

Theorem 1. Any injection of the latter 3 classes can't belong to the surjective mappings set.

Theorem 2. Necessary criterion of surjectivity of the injective mappings $\varphi: \ N \rightarrow N$ is of an asymptotic nature: $\lim(\varphi(n) : n) = 1$.

Theorem 3. There isn't a bijection between natural numbers set N and its proper subset $A \subset N$.

The concept of numerical sequence convergence is generalized in following way:

Definition 1. Numerical sequence (a) is termed as *a properly convergent sequence*, if

$$\lim(a_{n+1} - a_n) = 0. \tag{1}$$

This concept substantiates the existence of infinity hyper-real numbers. For example, both sequences (a) and (b) defined as $a_n = n^{1-\alpha}$, and $b_n = a_n(lnn)^{1-\alpha}, \ \alpha > 0$, satisfy condition (1).

Statement. So $(a_n), \ a_n = \sum_{p=1}^{n} p^{-1}, \ n \in N$, satisfies condition (1) then solution of an asymptotic equation $a_\infty{=}Arcsin(x_\infty)$ exists.

It is easy to prove the following statement by (1):

Theorem 4. A set of Cauchy's sequences includes unlimited ones.

The concept of numerical series defined more exactly makes it possible to prove

Theorem 5. The convergence of a numerical series (A) doesn't depend on permutation of $(A)'s$ addends.

Example. Let $(A) = \sum(-1)^{n+1}n^{-1} = A = \ln 2$. Series (B) was obtained from (A) by the following "procedure": q of sequential negative $(A)'s$ addends were put after p of sequential positive ones. The sequence (S_n) of partial $(B)'\mathbf{s}$ sums converges to number $S = \ln 2\sqrt{pq^{-1}}$. The sequence (r_n) of $(B)'\mathbf{s}$ residuals converges [2] to number $r = \ln\sqrt{p^{-1}q}$. Thus, $A = S + r$.

References

[1] Galilei G. *Selected Works: In 2 T.-Moscow: Science, 1964. T. 1.- 571 p. (In Russian)*

[2] Sukhotin A. M. *Alternative analysis principles: Study.-Tomsk: TPU Press, 2002.-43 p.*