The starting point of our work is the following remark given by S. P. Novikov [1]: "As Misha Gromov often explains in his lectures, Hyperbolic Geometry is visible from the infinity as one-dimensional one. Therefore, we may conclude, that for the discrete groups in 2D Lobachevski Plane with Noncompact Fundamental Domain of finite volume, Spectral Theory of the Laplace–Beltrami Operator should look in a sense 'similar' to the one on the graphs with k tails". We study here a more realistic model of the mentioned Spectral Theory considering a compact Riemannian 2D manifold X with k tails attached to it [2]. Like the spectrum of the automorphic Laplacian, the spectrum of the Laplace–Beltrami operator on such a "hybrid" manifold $\widehat{X}$ consists of 3 parts: (1) k-fold degenerated absolutely continuous spectrum; (2) at most $2k$ eigenvalues below the continuous spectrum; (3) eigenvalues imbedded in the continuous spectrum. At the generic boundary conditions determining the Laplace–Beltrami operator on $\widehat{X}$, the eigenvalues from (2) correspond to the poles of the scattering matrix on $\widehat{X}$, whereas the eigenvalues from (3) are just the multiple eigenvalues of the Laplace–Beltrami operator on X. Note that in the case of the spectral theory of automorphic functions, the problem of describing the imbedded eigenvalues is still open [3]. We describe the eigenfunctions corresponding to the eigenvalues of both the types (2) and (3) as well as the generalized eigenfunctions of the continuous spectrum, which have some properties similar to those for the Eisenstein series. Using a version of the Selberg trace formula from [4] we express the Selberg zeta function for X through the scattering matrix on $\widehat{X}$.

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