Asymptotic one phase soliton type solutions to perturbed Korteweg-de Vries equation

Valeriy Hr. Samoylenko

Korteweg-de Vries equation is one of the most known nonlinear partial differential equation and it is a fundamental equation of modern physics. It was deduced in 1895 in order to mathematically describe so-called solitary waves on water discovered by Jone Scott Russel in 1834. The secondary discovery of this outstanding equation in 1965 is connected with attempt to solve famous Fermi-Pasta-Ulam problem. At present it is appeared as object for studying in different fields of physics.

During the last 35 years a lot of papers was devoted to consideration of different properties of solutions to the Korteweg-de Vries equation, specifically, finding its solutions in exact form mainly via inverse scattering transform approach. On the other hand, while considerating processes with small variation of medium characteristics a problem of studying Korteweg-de Vries equation with varying coefficients and small parameters arises.

We consider Korteweg-de Vries equation with varying coefficients and small parameter at the higher derivative of the following form

$$u_{xxx} = a(x,\varepsilon)u_t + b(x,\varepsilon)uu_x. \tag{1}$$

Functions $a(x,\varepsilon), b(x,\varepsilon)$ are assumed to be represented as

$$a(x,\varepsilon) = \frac{1}{\varepsilon^n}\sum_{k=0}^{\infty} a_k(x)\varepsilon^k, \qquad b(x,\varepsilon) = \frac{1}{\varepsilon^n}\sum_{k=0}^{\infty} b_k(x)\varepsilon^k,$$

where $x \in \mathbf{R}^1$; $t \in (0;T)$; $a_k(x), b_k(x) \in C^\infty(\mathbf{R}^1), k \geq 0$; n is an integer.

Basing on small parameter approach we present algorithm of constructing asymptotical approximation for one phase soliton type solutions to problem (1). Specific peculiarity of the problem is a lot of additional problems connected with nonlinearity of unperturbed problem (when parameter $\varepsilon = 0$ in (1)), different type of asymptotics depending on integer n and necessity of determinating phase (curve of discontinuity) of a solution. We present also theorems on the order with that asymptotical solution satisfies the problem.