The Limit Analysis Method for Continuum Media

Igor A. Brigadnov
Department of Computer Science,North-Western State Technical University
Millionnaya 5, St. Petersburg, 191186, Russia
brigadnov@nwpi.ru

The limit analysis method for an estimation of the global stability of non-linear elastic and dielectric solids has been proposed in [1,2]. Here we examine the general limit analysis method for quasi-static problems of Continuum Mechanics. Namely, let a domain $\Omega \subset \mathbb{R}^n$ $(n \geq 1)$ be bounded with the Lipschitz boundary $\partial\Omega = \Gamma^1 \cup \Gamma^2$, where $\Gamma^1 \cap \Gamma^2 = \emptyset$ and $|\Gamma^1| > 0$. Let a physical state of a homogeneous continuum medium in Ω be described by the field $\mathbf{u} : \overline{\Omega} \to \mathbb{R}^m$ $(m \geq 1)$ with the zero trace on Γ^1. Let a work of external quasi-static influences $(\mathbf{f}, \mathbf{g}) \in L^\infty(\Omega, \mathbb{R}^m) \times L^\infty(\Gamma^2, \mathbb{R}^m)$ on the field $\mathbf{u}$ be described by the linear functional $L(\mathbf{u}) = \int_\Omega \mathbf{f}(\mathbf{x}) \cdot \mathbf{u}(\mathbf{x})\, d\Omega + \int_{\Gamma^2} \mathbf{g}(\mathbf{x}) \cdot \mathbf{u}(\mathbf{x})\, d\gamma$.

The initial limit analysis problem (LAP) is formulated in the following form:

$$t_* = \inf \left\{ \int_\Omega |\nabla \mathbf{u}(\mathbf{x})|\, d\Omega : \ \mathbf{u} \in V, \ L(\mathbf{u}) = 1 \right\}, \tag{1}$$

$$V = \left\{ \mathbf{u} \in W^{1,1}(\Omega, \mathbb{R}^m) : \ \mathbf{u}_{|_{\Gamma^1}} = \mathbf{0} \right\}.$$

and the dual LAP has the form

$$\tau_* = \inf \left\{ \|\mathbf{S}\|_\infty : \ \mathbf{S} \in G \right\}, \tag{2}$$

$$G = \left\{ \mathbf{S} \in L^\infty(\Omega, \mathbb{R}^{n\times m}) : \ \int_\Omega (\nabla \cdot \mathbf{S} + \mathbf{f}) \cdot \mathbf{u}\, d\Omega = 0, \ \int_{\Gamma^2} (\mathbf{n} \cdot \mathbf{S} - \mathbf{g}) \cdot \mathbf{u}\, d\gamma = 0, \ \forall \mathbf{u} \in V \right\},$$

where symbol $\mathbb{R}^{n\times m}$ denotes the space of real $n \times m$ matrices. The main relation $t_* \tau_* = 1$ is proven.

It is demonstrated that the initial LAP (1) needs a relaxation, but the appropriate fully relaxed problem has no clear physical interpretation. On the other hand, the dual LAP (2) has a clear physical interpretation.

For the 2-D initial LAP the partial relaxation based on the discontinuous finite-element approximation (FEA) is proposed, and for the dual LAP the standard piecewise linear continuous FEA is used. The numerical results show that the proposed limit analysis method has a qualitative advantage over classical techniques of estimation of the global stability of a continuum medium in Ω.

References

[1] I.A. Brigadnov. The limited static load in finite elasticity. In Al Dorfmann and A. Muhr, editors, *Constitutive Models for Rubber*, pages 37–43, Rotterdam, A.A.Balkema, 1999.

[2] I.A. Brigadnov. Numerical analysis of dielectrics in powerful electrical fields. *Computer Assisted Mech. Eng. Sci.*, 8: 227–234, 2001.