The twistorial interaction principle from orthosymplectic graded Lie algebras

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Cartan's triality principle, based on the group SO(8) and its double covering Spin(8), has been applied in almost all recent physical theories. For instance, Spin(8) invariance is the unique symmetry in the light-cone gauge under Majorana-Weyl conditions [1], and in (1+9) dimensions the massless little group is SO(8). Only in the specific dimension eight, bosons and fermions have the same number of degrees of freedom. This generalized supersymmetry comes from the triality principle, which asserts that bosons and fermions are equivalent under the triality map, viz., an order-3 automorphism that cyclically interchanges vector and (inequivalent) spinor representations. The triality map is based on the (commutative and non-associative) Chevalley product [2].

The triality principle has been shown to be of extreme importance in D=10 supersymmetric theories. It is known that, aside from IIB superstring theories, there is no SO(8) spinor decomposition preserving SO(8) symmetry. In this situation one can introduce distinct coordinates and conjugate momenta [1] only if Spin(8) symmetry is broken by a Spin(6)$\times$ Spin(2) $\simeq$ SU(4)$\times$ U(1) subgroup of Spin(8).

We describe this subgroup as the isotropy subgroup on Pin(8) at a pure spinor, and we link the triality principle, through the structure of pure spinors, to Penrose's twistor theory. We construct a generalization of Penrose's flagpole [3] and we prove that a twistor is an algebraic spinor associated to the Dirac-Clifford algebra $\mathbb{C} \otimes C\ell_{1,3}$ (the Clifford algebra over Minkowski (1+3)-spacetime), using one lower dimension than in [4]. Our viewpoint sheds new light on twistor theory, for it show that we can identify the twistor fiber with the homogeneous space O(8)/(Spin(6)$\times$ Spin(2)).

We finally describe the interaction principle, which generalizes the triality principle via orthosymplectic graded Lie algebras [5]. Using the Chevalley product we relate the algebraic pure spinor approach to the Wess-Zumino superfield formalism [6].

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