Spectral properties of weakly coupled waveguides and layers

Popov I.Yu.
Department of Higher Mathematics,
Institute of Fine Mechanics and Optics
Sablinskaya, 14, St.-Petersburg, 197101, Russia

Spectral problems for the Dirichlet (semiconductor nanostructures) and Neumann (metallic nanostructures) Laplacians in two-dimensional strips (three-dimensional layers) of widths d_+, d_-, $d_+ > d_-$ coupled through small openings $a\omega_i, i = 1, ...n$, (a is small parameter) is considered. Method of matching of asymptotic expansions of solutions of boundary value problems is used.The asymptotics (in a) of a bound state λ_a close to the threshold π^2/d_+^2 is obtained The asymptotics of a resonance close to $n-th$ threshold is obtained. The case of two identical waveguides (layers) is considered too. Asymptotics of bands for the case of periodic system of coupling windows (period L) for two 2D and 3D waveguides is constructed. It is shown that there is a gap for suffisiently small a. For the case of layers coupled through singly (Λ_1) and doubly (Λ_2) periodic system of windows there is no gap, and the asymptotics of the lower bound for the continuous spectrum is obtained.

A system of three two-dimensional Dirichlet strips $\Omega_1, \Omega_2, \Omega_3$ coupled through small windows is considered. If the system is symmetric in respect to the centreline of the waveguide Ω_2 and $d_2 > d_1 = d_3 > d_2/2$ (d_i is the width of Ω_i) there is an eigenvalue embedded in the continuous spectrum of the corresponding Dirichlet Laplacian. The asymptotics of the eigenvalue (in the width of the windows) is obtained. If the symmetry is broken ($d_1^2 = d_3^2 + \xi, \xi > 0$,) the eigenvalue transforms to the resonance (quasi-eigenvalue). The asymptotics of the resonance is constructed. We look for the transformation process when $\xi \to 0$. The behaviour of the resonance is described. Possible application of the phenomena in nanoelectronics is discussed.

The initial terms of the asymptotic expansion of quasi eigenvalue k_a^2 close to the threshold $\frac{\pi^2}{d_+^2}$ of the Neumann Laplacian for two strips coupled through small window of width $2a$ (or finite number of windows) are found. The asymptotics of the resonant band close to the threshold in the case of periodic system of coupling windows is obtained. The asymptotics of the resonance for the case of layers with the Neumann boundary condition coupled through small windows is constructed.

The system of two-dimensional dielectric waveguides separated by thin high-contrast dielectric strip with small window is considered. It is proved that there exists an eigenvalue below the threshold and variational estimates of the eigenvalue are found. The estimates of the corresponding band are obtained in the case of periodic system of coupling windows. The possibility of gap existence is shown. Possible applications to optical fibers systems are discussed.