Dualities between torsion-free abelian groups of finite rank and their endomorphism rings

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Let $\mathcal{A}$ be a particular class of torsion-free abelian groups, the so-called *almost completely decomposable groups* (ACD-groups), having the regulator A, a fully invariant completely decomposable subgroup of finite rank. According to the classical definition of ACD-groups, for any $X \in \mathcal{A}$ there exists a finite set of primes $P = P(X)$ such that $X = \sum_{p \in P} X_p$ with $X_p \in \mathcal{A}$ and X_p/A is a p-primary finite group. It was shown in [1] that for any group $X \in \mathcal{A}$ its subgroups, members of $\mathcal{A}$, and their endomorphism rings form two anti-isomorphic Boolean algebras with respect to the set theory operations $+$ and $\cap$. Since torsion-free abelian groups have complicated structures with non-isomorphic direct decompositions, they are traditionally classified up to near-isomorphism ($\cong_{nr}$), an equivalence, which is weaker than isomorphism ($\cong$), but preserves the decomposability properties. Our consideration of ACD-groups in the dual connection with their endomorphism rings leads to the extension of some results from ACD-groups to the rings, see [1-2].

Theorem 1. Let $X, X' \in \mathcal{A}$. If $X \cong_{nr} X'$ then $\operatorname{End} X$ and $\operatorname{End} X'$ are nearly isomorphic as abelian groups.

Theorem 2. Let $X, X' \in \mathcal{A}$ and X/A, X'/A be cyclic groups. Then $X \cong_{nr} X'$ if and only if $\operatorname{End} X$ and $\operatorname{End} X'$ are isomorphic as rings.

References

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