ABSTRACT

In this abstract G denotes a non-trivial finite group and $c.d.(G)$ denotes the set of degrees of the irreducible characters of G. In his well-known book, Martin Isaacs proved in that if the commutator subgroup G' is the unique minimal normal subgroup of a finite solvable group G, then $c.d.(G) = \{1, f\}$ for some $f > 1$. Moreover, groups G satisfying the above condition are shown to be either p-groups of certain type or Frobenius groups of certain type. In our paper finite solvable groups G with G' being a minimal normal subgroup, are described. In particular, it is shown that the irreducible characters of such groups have exactly two degrees. Using this result, non-abelian groups with two character degrees are investigated, including the characterization of such groups if $G' \cap Z(G) = 1$. If $Z(G) = 1$, then it is shown that the irreducible characters of G have exactly two degrees if and only if the conjugacy classes of G are of exactly three sizes.

References

E.A. Bertram, M. Herzog and A. Mann "On a graph related to conjugacy classes of groups" Bull. London. Math. Soc. vol 22 (1990), 569-575.

P.X. Gallagher "The number of conjugacy classes in a finite group" Math.Z. vol 118 (1970), 175-179.

B. Huppert "Character theory of finite groups" Walter de Gruyter Berlin (1998).

I.M. Isaacs "Character theory of finite groups" Dover Publications, Inc. New-York (1994).