The Mahler measure of an algebraic number α is defined by the formula $M(\alpha) = a\prod_{j=1}^{d}\max\{1,|\alpha_j|\}$, where $\alpha_1 = \alpha, \alpha_2, \ldots, \alpha_d$ are the conjugates of α over $\mathbf{Q}$ and a is the leading coefficient of its minimal polynomial in $\mathbf{Z}[x]$. There are many open questions concerning the set of all Mahler measures $\mathcal{M}$ of algebraic numbers. For instance, D.H. Lehmer asked in 1933 whether there are elements of the set $\mathcal{M}$ greater than 1 and smaller than 1.17.

In our recent paper "The values of Mahler measures" (with J.D. Dixon, to appear in "Mathematika") we show that if β is a unit then one can determine whether β belongs to $\mathcal{M}$ or not by a finite computation. Namely, if $\beta \in \mathcal{M}$ then $\beta = M(\alpha)$ with some α of degree at most $\binom{n}{[n/2]}$, $n = \deg\beta$, lying in the normal closure of $\mathbf{Q}(\beta)$ over $\mathbf{Q}$. We also show that the set $\mathcal{M}$ is not a multiplicative semigroup. In the paper "Mahler measures generate the largest possible groups" (to appear in "Math. Res. Lett.") we show that $\mathcal{M}$ is not an additive semigroup. It is also shown that the set $\mathcal{M}$ is very rich in the following sense. Firstly, every real positive algebraic number can be written as a quotient of two elements of $\mathcal{M}$. Secondly, every real algebraic integer γ can be written as $\gamma = \sum_{i=1}^{4} k_i m_i$, where $m_1, m_2, m_3, m_4 \in \mathcal{M}$ and $k_1, k_2, k_3, k_4 \in \mathbf{Z}$.