

An almost forgotten Gauss' little geometry gem is as follows. Consider a convex pentagon 01234 of area A . To compute A , you don't have to triangulate, dissect or integrate but just go around the pentagon and measure its vertex triangles. Let their areas be (0) , (1) , (2) , (3) , (4) , and let

$$\begin{aligned}c_1 &= (0) + (1) + (2) + (3) + (4), \\c_2 &= (0)(1) + (1)(2) + (2)(3) + (3)(4) + (4)(0).\end{aligned}$$

In fact, c_1 and c_2 are the first and second cyclic symmetric functions of (0) , (1) , (2) , (3) , (4) . The Gauss pentagon formula is:

$$A^2 - c_1A + c_2 = 0.$$

In particular, if the pentagon has all vertex triangles of area a , then $A = \sqrt{5}\varphi a$, where φ is the golden ratio.

We give several proofs of this beautiful formula, relate it to Monge and Ptolemy formulas, and extend it to hexagons, discuss affine regular polygons etc.