Abstract

Following Nakamura's definition of G-Hilbert scheme $G-Hilb\mathbb{A}^n$, Craw and Reid provide a much more easy algorithm for computing $G-Hilb\mathbb{A}^3$ for G a diagonal subgroup of $SL_3(\mathbb{C})$. This uses a reduction to the 2-dimensional methods by means of Hirzebruch-Jung continued fractions and a division of the junior simplex into regular triangles. Each regular triangle corresponds into the $G-$Hilbert sheme to a $G-$cluster and, in the polynomial ring of $\mathbb{A}^3$, to a certain subset of monomials called G-graph (see [N] for definitions of $G-$graph and $G-$Hilbert scheme). The nice description given in [CR] shows also that a $G-$graph has the property of "tessalting" the plane .

Even if it is well-known, by [BKR], that for any subgroup of $SL_n(\mathbb{C}), n = 2, 3$ the G-Hilbert scheme of $\mathbb{A}^n$ is a crepant resolution of singularities of the quotient singularity $\mathbb{A}^n/G$, the decription of this sheme in terms of $G-$graphs in the case of a non-abelian group G is rather difficult. R. Leng (see [L]) defines a similar notion of $G-$graph for G a binary-dihedral goup and computes the $G-Hilb\mathbb{A}^2$ in this case.

This poster tries to explain what are the results to be expected in the case of a trihedral group. Namely to give some answers to questions as: is it true that each $G-$cluster corresponds to a unique set of [quasi]monomials of the polynomial ring of $\mathbb{A}^3$? How to define a $G-$graph? How should one translate the tessaltion property of [CR] in this case? For a good definition of $G-$graph in this case, is it enough to ask for the translation properties or there should be more conditions to besatified (such as, for example, the invaraince by 120 degrees rotations)?

References

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