The study of the total squared mean curvature functional for immersed surfaces in R^3, $W(f)$ was initiated by G. Thomsen and W. Blaschke in 1923. Critical points of $W(f)$ are usually known as Willmore surfaces. T.J. Willmore started in 1968 the study of the so called Willmore problem (the determination of the infimum of $W(f)$ among all immersions of a compact surface of a given topological genus) and posed his famous Willmore conjecture which have been topics of intense activity during the last decades [5]. In the early seventies, B-Y Chen [3] extended the Thomsem-Willmore functional to any submanifold M of any Riemannian manifold. He defined the Chen-Willmore functional by

$$CW(M) = \int_M (\alpha^2 - \tau_e)^{(n/2)} dv, \tag{1}$$

α and τ_e being the mean curvature and the extrinsic scalar curvature of M, respectively. It is conformal invariant and its critical points are known as Chen-Willmore submanifolds. In this poster we review some author's recent work on the subject. We describe a new method to obtain Willmore-Chen submanifolds in spaces endowed with warped product metrics and fibers being a given homogeneous space [1], [2]. The main point is that we can relate this variational problem to that of generalized elastic curves in the conformal structure on the base space. We also obtain some applications of our method. For example, we show that closed Chen-Willmore rotational hypersurfaces of non-negative curved real space forms are shaped on closed hyperelastic curves of the hyperbolic plane. Then, we study the variational problem associated to this class of curves, proving that there exist a rationally dependent family of closed solutions. They give rise to the first non-trivial examples of Chen-Willmore hypersurfaces in real space forms. Finally, we discuss a numerical confirmation of the generalized Willmore conjecture.

References

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