B.Y. Chen [2] established a sharp inequality for the warping function of a warped product submanifold in a Riemannian space form in terms of the squared mean curvature. For a survey on warped product submanifolds we refer to [3].

In [5], we established a corresponding relationship between the warping function f (intrinsic structure) and the squared mean curvature and the holomorphic sectional curvature (extrinsic structures) for warped product submanifolds $M_1 \times_f M_2$ in any complex space form.

In the present paper, we investigate warped product submanifolds in quaternion space forms $\widetilde{M}^m(4c)$. We obtain different estimates of the mean curvature in terms of the warping function, whether $c < 0$, $c = 0$ and $c > 0$, respectively. Equality cases are considered and certain examples are given.

As applications, we derive obstructions to minimal warped product immersions in quaternion space forms. For example, the non-existence of minimal proper warped product submanifolds $M_1 \times_f M_2$ in the m-dimensional quaternion Euclidean space $\mathbf{Q}^m$ with M_1 compact is proved.

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