Representations of $(1,1)$-knots

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Abstract

A knot K in a 3-manifold N^3 is called a $(1,1)$-*knot* if there exists a Heegaard splitting of genus one $(N^3, K) = (H, A) \cup_\psi (H', A')$, where H and H' are solid tori, $A \subset H$ and $A' \subset H'$ are properly embedded trivial arcs and $\psi : (\partial H', \partial A') \to (\partial H, \partial A)$ is an attaching homeomorphism. Obviously, N^3 turns out to be a lens space (possibly $\mathbf{S}^3$). In particular, the family of $(1,1)$-knots contains all torus knots and all two-bridge knots in $\mathbf{S}^3$. The topological properties of $(1,1)$-knots have recently been studied in several papers from different points of view (see references in [2]).

We develop two different representations of $(1,1)$-knots and study the connections between them.

The first is algebraic: every $(1,1)$-knot is represented by an element of the pure mapping class group of the twice punctured torus $PMCG_2(T)$, where $T = \partial H$ (see [1, 2]). Moreover, there is a surjective map from the kernel of the natural homomorphism $\Omega : PMCG_2(T) \to MCG(T) \cong SL(2,\mathbb{Z})$, which is a free group of rank two, to the class of all $(1,1)$-knots in a fixed lens space.

The second is parametric: using the results of [4] and [3], every $(1,1)$-knot can be represented by a 4-tuple (a, b, c, r) of integer parameters.

The above representations are explicitly obtained in many interesting cases, including two-bridge knots and torus knots.

References

[1] A. Cattabriga and M. Mulazzani: *Strongly-cyclic branched coverings of (1,1)-knots and cyclic presentations of groups.* Math. Proc. Cambridge Philos. Soc. **135** (2003), 137-146.

[2] A. Cattabriga and M. Mulazzani: *(1,1)-knots via the mapping class group of the twice punctured torus*. Adv. Geom. **4** (2004), 263-277.

[3] A. Cattabriga and M. Mulazzani: *All strongly-cyclic branched coverings of* $(1,1)$*-knots are Dunwoody manifolds*. J. London Math. Soc., to appear, arXiv:math.GT/0309298

[4] L. Grasselli and M. Mulazzani: *Genus one 1-bridge knots and Dunwoody manifolds*. Forum Math. **13** (2001), 379-397.