Numerical solution of differential algebraic stiff systems.

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Many practical problems are described by systems of both differential and algebraic equations. For example, while simulating non-stationary processes in the electrical circuits algebraic Kirchhoff's laws are taken into account, the PDE system for gas dynamics is supplemented by algebraic condition equations. One or more differential equations in a system are possible implicit algebraic. In general statement of the problem is $M\frac{d\vec{u}}{dt}=f\left(\vec{u},t\right)$, where $\det M=0$ is the singular matrix.

Additional difficulty for numerical solution is following. All investigated problems are stiff. Stiff system is a problem in which two or more physical processes have strongly distinguishing temporal characteristics. For example while simulating processes in electrical circuits the frequency of an alternating current is much greater, than frequency of amplitude modulation; in mathematical modeling of chemically reacting gas flows tens chemical reactions with quite different speeds are taken into account. Stiff systems are requiring the development of the special difference scheme, with increased requirements to the stability.

In this paper the sets 2-stages difference schemes of Rosenbrock are tested. By results of a theoretical investigating two $L1$-stable schemes of accuracy $O\left(\tau^2\right)$ and A - stable one of accuracy $O\left(\tau^3\right)$ are recommended. Both autonomous $f\left(\vec{u},t\right)=f\left(\vec{u}\right)$ and non-autonomous problems are considered. Simulation of the transistor amplifier and gas flows was curried out to illustrate possibility of new methods. Numerical results confirm the effectiveness, high accuracy and suitability for stiff systems of these schemes.

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