Ergodic and spectral properties of Lagrangian and Hamiltonian dynamical systems and their adiabatic perturbations.

Anatoliy K. Prykarpatsky
Dept. of Nonlinear mathematical Analysis at the IPPMM of NAS of Ukraine
and
Dept. of Applied mathematics at the AMM Univesrity of Science and Technology, Krakow, Poland

Any Lagrangian function on a closed finite-dimensional manifold M, when depending 2π- periodically on the evolution parameter generates so called Lagrangian flow. Its related group of diffeomorphisms na $T(M) \times \mathbb{S}^1$ makes it possible to constuct the set of normed (probabilistic) invariant measures on $T(M) \times \mathbb{S}^1$. The latter appears to be a convex set completely characterized by means of so called extreme points being at the same time due to a result of J. Mather ergodic measures of the Lagrangian flow under regard. On the other hand, there exists a natural mapping from the space of all invarian mesure space mentioned above into the first homology group $H(M;\mathbb{R})$ of the manifold M via a well known Mather's construction, whose image is exactly the measure homology of our Lagrangian system. Its properties prove to be very very important for detecting the corresponding ergodic mesures, making use a new tool of its studing related with so called Legendrian transformations and Poincare -Cartan invariants. Moreover in the case when our Lagrangian funcion depends adiabatically on a small parameter $\varepsilon \downarrow 0$ through the expession $\varepsilon t \in \mathbb{R}/2\pi\mathbb{Z}$, the suitable application of the Legendrian transformation together with the technique of Poincare -Cartan invariants makes it possible to investiagte the existence and properties of so called adiabatic invariants and the corresponding limiting ergodic measures on $T(M) \times \mathbb{S}^1$. These same properties are studied simultaneously making use also of the theory of spectral invariants applied to the generator of the corresponding Hamiltonian flow on the symplectic phase space $T^*(M)$.