

**Section number:** 08

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**On Bounded Solutions of Second Order Nonautonomous Nonlinear Differential Equations**

The problem

$$u'' = f(t, u, u'), \quad (1)$$

$$\sup\{|u(t)| : t \in \mathbb{R}\} < +\infty \quad (2)$$

is considered, where  $f : \mathbb{R}^3 \rightarrow \mathbb{R}$  is a continuous function.

**Definition.** Suppose  $\sigma_i : \mathbb{R} \rightarrow \mathbb{R}$  ( $i = 1, 2$ ) are continuous functions such that

$$\sigma_1(t) \leq \sigma_2(t) \text{ for } t \in \mathbb{R}. \quad (3)$$

We say that the function  $f$  belongs to the class  $\mathcal{B}(\sigma_1, \sigma_2)$  if there exists a continuous function  $\rho : \mathbb{R} \rightarrow [0, +\infty[$  such that for any continuous function  $\delta : \mathbb{R} \rightarrow [0, +\infty[$ , an arbitrary solution of the problem

$$u'' = \delta(t)f(t, u, u'), \quad \sigma_1(t) \leq u(t) \leq \sigma_2(t) \text{ for } t \in \mathbb{R}$$

admits the estimate  $|u'(t)| \leq \rho(t)$  for  $t \in \mathbb{R}$ .

The following theorem is proved.

**Theorem.** *Let there exist bounded on  $\mathbb{R}$  lower and upper functions  $\sigma_1$  and  $\sigma_2$  of Eq. (1) satisfying condition (3). Let, moreover,  $f \in \mathcal{B}(\sigma_1, \sigma_2)$ . Then problem (1), (2) has at least one solution.*

From this theorem several effective conditions for the solvability of problem (1), (2) are obtained.

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