Periodic Solutions of Impulsive Matrix Differential Equation

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We consider a differential matrix equation with bilinear main part

$$dX/dt = A(t)X - XB(t) + \sum\nolimits_k [D_k]\delta(t-t_k)X + F_\delta(t), \tag{1}$$

where $F_\delta(t) = F(t) + \sum_k \widetilde{F}_k \delta(t-t_k)$, $[D_k]Z = D_k Z \widetilde{D}_k$; $A(t), D_k \in R^{n\times n}$; $B(t), \widetilde{D}_k \in R^{m\times m}$; $X, F, \widetilde{F} \in R^{n\times m}$; $\delta(t-t_k)$ is Dirac's measure. The general solution of that equation deal with the solution of difference operator equation.

Proposition. The solution of equation (1) is the distribution with continuous derivative at any interval (t_k, t_{k+1}) $(-\infty < k < \infty)$. It is defined by equalities

$$\begin{array}{c} X_t(t_0, X_0) = [\Omega^t_{t_k^+}]C_k + \int_{t_k^+}^t [\Omega^t_\tau]F(\tau)d\tau,\ t \geq t_0 \\ C_k = [\alpha_k]C_{k-1} + \beta_k, \quad k = 1, 2, ..., \end{array} \tag{2}$$

where $[\Omega^t_\tau]$ is the evolutionary operator of the homogeneous equation (1) at interval (t,τ), $[\Omega^t_\tau]Z = \underset{A}{\Omega}{}^t_\tau Z \underset{B}{\Omega}{}^\tau_t$, $[\alpha_k] = ([I]+[D_k])[\Omega^{t_k^-}_{t_{k-1}^+}]$, $\beta_k = ([I]+[D_k])\int_{t_{k-1}^+}^{t_k^-}[\Omega^{t_k^-}_\tau]F(\tau)d\tau + \widetilde{F}_k$.

The formula (2) is equivalent to the formula which have been given in the monograph [1] but is more convenience for numerical calculations. One can easy to see that if $F(t) \equiv 0$, $[D_k] \equiv 0$, then $[\alpha_k] = [\Omega^{t_k^-}_{t_{k-1}^+}]$, $\beta_k = \widetilde{F}_k$ and the formulas (2)-(5) are transforming into the formulas that was given in [2].

The equation (1) is periodic with period ω if the matrix-valued functions $A(t)$, $B(t)$, $F(t)$ are ω-periodic, and there is a natural number p such that $D_{k+p} = D_k$, $\widetilde{D}_{k+p} = \widetilde{D}_k$, $\widetilde{F}_{k+p} = \widetilde{F}_k$ for all k, and $t_{k+p} = t_k + \omega$. The difference equation in (2) then will be periodic with the period equal p. It is interesting for application the case when $p = 1$, $[D_k] = constant$, $\widetilde{F}_k = constant$, $F(t) = 0$. This impulsive differential problem is called stroboscopic problem [3].

In our report we consider the properties of periodic solution, the conditions of its stability and conditions of convergence the approximate solution if iterative method will be applied.

References.

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