We solve numerically the following problem, which is defined in $Q_T = (0,1)\times(0,1)\times(0,T]$:

$$\begin{cases} \dfrac{\partial u}{\partial t} = \sum\limits_{\alpha=1}^{2} \dfrac{\partial}{\partial x_\alpha}\left(k_\alpha(X,t)\dfrac{\partial u}{\partial x_\alpha}\right) - q(X,t)u + f(X,t), \quad (X,t)\in Q_T\,, \\ u(0,x_2,t) = \mu_1(x_2,t), \quad u(1,x_2,t) = \mu_2(x_2,t), \quad x_2\in[0,1],\ t>0\,, \\ u(x_1,0,t) = \mu_0(t)\mu_3(x_1), \quad u(x_1,1,t) = \mu_4(x_1,t), \quad x_1\in[0,1],\ t>0\,, \\ u(x_1,x_2,0) = u_0(x_1,x_2), \quad (x_1,x_2)\in[0,1]\times[0,1]\,, \end{cases} \tag{1}$$

with the additional integral condition:

$$\int_0^1 \int_0^{d(x_1)} \rho(x_1,x_2)u(x_1,x_2,t)\,dx_2\,dx_1 = M(t), \quad t\in[0,T]\,. \tag{2}$$

A splitting scheme is used to approximate the given problem. It is proved that the proposed scheme has a solution for less restrictive conditions than the explicit schemes, which are proposed in [1]. One economical algorithm is presented for finding a solution of the obtained system of linear equations.

A modification of the explicit Euler scheme from [1] is also presented. We have proved that it is sufficient to approximate the integral condition by the second order accuracy quadrature formula in order to get the second oder accuracy of the discrete solution.

Results of numerical experiments are presented, which confirm the obtained theoretical results.

References

[1] B.J. Noye, M. Dehghan. Numer. Meth. for PDE. **15** (1999), 521–534.