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Text of abstract:

In our poster, we shall present the main ideas and techniques we employed to prove three characterizations of the Euclidean sphere, which involve certain boundary value problems for the heat and porous medium equations.

A bounded heat conductor Ω has (non-zero) constant initial temperature while, at each time, its boundary is kept at zero temperature. This physical situation can be modelled as an initial-boundary value problem for the heat equation. The following symmetry result settles and improves a conjecture of M.S. Klamkin: *if a heat conductor contains a stationary isothermic surface, then it must be a ball* [Magnanini-Sakaguchi, Ann. Math. 156 (2002).] (A *stationary isothermic surface* is a spatial level surface of temperature which is invariant in time.)

A gas flows into an initially empty container Ω from the container's walls $\partial\Omega$, where the gas density is always kept constant. (Here, the porous medium equation is at stake.) Choose a number $R > 0$; *if the gas content of every closed ball $B(x, R)$ (touching $\partial\Omega$ only at one point) does not depend on x for each given time, then the container Ω must be a ball.*

A domain Ω is *uniformly dense at* $\partial\Omega$ if, for every $x \in \partial\Omega$ and $r > 0$, the volume of the intersection of Ω with the ball $B(x, r)$ does not depend on x, but possibly on r. A domain is uniformly dense at $\partial\Omega$ if and only if $\partial\Omega$ is an isothermic surface for a certain initial value problem for the heat equation. We prove that *bounded uniformly dense domains must be balls.*

In a final section, we will consider the case in which the relevant sets Ω are no longer bounded. For instance, with the aid of a theorem of J. Nitsche [Analysis 15, (1995)], we characterize three-dimensional uniformly dense domains.