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Let $\mathbb{R}^d$ be a bounded domain with smooth boundary Γ and $\mathcal{A}(x, D)$ be a differential expression of order $2m$ on $\Omega \cup \Gamma$ with smooth coefficients. We assume that $\mathcal{A}(x, D)$ is a formal self-adjoint strongly elliptic differential expression and its principal symbol is positive. Let denote by $(\mathcal{A}, \Omega)$ a set of all self-adjoint realization of the formal differential operator $\mathcal{A}(x, D)$ in the Hilbert space $L_2(\Omega)$. A distribution of eigenvalues for a operator $A \in (\mathcal{A}, \Omega)$ is studied under additional assumptions of ellipticity $(\mathcal{E})$ or coerciveness $(\mathcal{C})$. Precise asymptotic formulae for the counting functions $N_{\pm}(\lambda, A) := card\{k : \pm\lambda_k(A) \in (0, \lambda)\}$ are found (cf. [1], [2], [3]).

THEOREM 1. *If operator $A \in (\mathcal{A}, \Omega)$ is positive and satisfies the condition*

$$Q(A) = D(A^{1/2}) \subset H^m(\Omega), \qquad (\mathcal{C})$$

then the Weyl's asymptotic formula

$$N_+(\lambda, A) = w\lambda^{d/2m} + O(\lambda^{(d-1)/2m}), \quad as \quad \lambda \to \infty \qquad (*)$$

(with standard coefficient $w = w(\mathcal{A}', \Omega) > 0$) is valid.

REMARK. The condition $(\mathcal{C})$ is fulfilled if $A > 0$ and

$$D(A) \subset H^{2m}(\Omega). \qquad (\mathcal{E})$$

THEOREM 2. *If operator $A \in (\mathcal{A}, \Omega)$ is non-semibounded and the condition $(\mathcal{E})$ is fulfilled then*

$$N_-(\lambda, A) = O(\lambda^{(d-1)/2m}), \quad as \quad \lambda \to \infty \qquad (**)$$

and asymptotic formula $(*)$ holds.

The results are precise in the following sense. We can not replace "O" with "o" in formulae $(*)$ and $(**)$.

??1 Agmon S. Asymptotic formulas with remainder estimates for eigenvalues of elliptic operators Arch. Ration. Mech. Analysis 1968 28 3 165-183 **??**2 Browder F. Asymptotic distribution of eigenvalues and eigenfunctions for non-local elliptic boundary value problems I Amer. J. Math. 1965 87 1 175-195 **??**3 Brüning J. Zur Abstratzung der Spectralfunktion elliptisher Operatoren Math. Z. 1974 137 1 75-85