$O(3)$-PARTIALLY INVARIANT SOLUTION OF MAGNETOHYDRODYNAMICS

Sergey Golovin
Lavrentyev Institute of Hydrodynamics SD RAS
Novosibirsk 630090, Russia
e-mail: sergey@hydro.nsc.ru

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The group $O(3)$ of rotation in three-dimensional space is a part of invariance group for most of mathematical models of continuous media. It is the reflection of the assumption of the space isotropy, which is commonly used in formulation of the mathematical problem. The $O(3)$ group, as any other continuous symmetry group of the system of PDEs, gives a source of exact solutions of the observed model. The classical rotationally invariant solution of the system of differential equations is the solution where all sought functions depend on radial coordinate only. From group theoretical point of view [1, 2] rotationally invariant solution is a singular $O(3)$-invariant solution. The nonsingular $O(3)$-invariant solution do not exist since the set of its invariants does not cover all sought functions (only two of three components of fluid's velocity vector field can be derived from the invariants). However, one can sought for $O(3)$-partially invariant solution [1].

The main problem in construction of partially invariant solution is that it is defined by the overdetermined system of PDEs, which is need to be completed to involution. This problem was successfully solved by Ovsiannikov [3] for $O(3)$-partially invariant solution of the ideal compressible and incompressible fluid models.

In present work we observe $O(3)$-partially invariant solution for ideal magnetohydrodynamics. This case is more complicated in comparison with ideal fluid since it is simultaneously involves two vector fields: fluid velocity and magnetic field. In the spherical coordinate system (r, θ, φ) the representation of solution has the form

$$\begin{aligned} &U = U(t,r), \quad M = M(t,r), \quad H = H(t,r), \quad H_\tau = H_\tau(t,r), \\ &\sigma = \sigma(t,r) + \omega(t,r,\theta,\varphi), \quad p = p(t,r), \quad \rho = \rho(t,r). \end{aligned} \tag{1}$$

Here p and ρ are pressure and density. The velocity and magnetic vector fields are characterized by its normal and tangential to the spheres $r = \text{const}$ components. Normal components of the velocity and magnetic fields are U and H. The tangential components of these vector fields are correspondingly specified by its modules M and H_τ and derivation from meridian angles ω and σ.

It is proved that the irreducible solution (function ω is determined with functional arbitrariness) exists only in the following cases

1^0. $H_\tau = 0$ — pure radial magnetic field;

2^0. $M = 0$ — pure radial gas flow;

3^0. $\sigma = \omega$ — identical derivation angles of the tangential component of the vector fields $\mathbf{u}$ and $\mathbf{H}$.

In all cases the function ω is determined with arbitrariness in one function of one argument. The main difference with ideal fluid model is that the system for invariant functions (which depend on t and r only) is also overdetermined. The latter is completed to involution. The implicit formulas of solution are given.

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