Let B_ρ denote the ball of radius ρ in $\mathbb{R}^N$ with $N \geq 2$. Let $1 < p < \infty$ be fixed and let $1 < q < p^*$ where p^* is the critical trace exponent. The best constant $S_q(\rho)$ in the trace inequality is characterized by

$$S_q(\rho) = \inf_{u \in W^{1,p}(B_\rho)} \frac{\int_{B_\rho} |\nabla u|^p + |u|^p \, dx}{\left(\int_{\partial B_\rho} |u|^q \, d\sigma \right)^{p/q}} \tag{1}$$

This infimum is reached by a function u which has definite sign. A natural question is whether u is a radial or a nonradial function. This problem has been studied in [1] and [2] in the case $p = 2$. Amongst other results the authors of these papers show that if ρ is sufficiently large then u is nonradial, whereas if ρ is sufficiently small then u is radial. We consider the more general setting $1 < p < \infty$ and we consider also the dependence of the radialicity of u on the parameter q. We extend many of the results known for the case $p = 2$. We also prove various results which are new even in the case $p = 2$. We show that there exists a radial function $u_0 > 0$ in $\mathbb{R}^N$, independent of q, such that any radial minimizer is a multiple of u_0. We show that if ρ is sufficiently large then there is no radial minimizer for (1), as in the case $p = 2$. Now let $\rho \mapsto Q(\rho)$ be defined by

$$Q(\rho) = \frac{1}{\lambda_1(\rho)^{p/(p-1)}} \left(1 - (N-1) \frac{\lambda_1(\rho)}{\rho} \right) + 1 \tag{2}$$

where $\lambda_1(\rho)$ is the first eigenvalue in an associated eigenvalue problem. We show that if $q > Q(\rho)$ then there is no radial minimizer for (1). Lastly we give numerical results suggesting that the converse is true: If $q \leq Q(\rho)$ then any minimizer is radial. This work has been submitted for publication as part of a joint paper with Enrique Lami Dozo.

References

[1] M. del Pino and C. Flores. Asymptotic behavior of best constants and extremals for trace embeddings in expanding domains. *Commun. in P.D.E.*, 26(11):2189–2210, 2001.

[2] J. Fernandez Bonder, E. Lami Dozo, and J. D. Rossi. Symmetry properties for the extremals of the Sobolev trace embedding. *To appear in Ann. Inst. Henri Poincaré.*