Equations of the vortex motion in the rotating Bose-Einstein condensate

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Abstract

We consider a Bose-Einstein condensate placed in a trap potential field rotating around fixed axis z (2D problem). Since the wave function u vanishes at infinity, we consider the problem in the *boundary domain*. The *heat flow equation* is studied.

We construct the vortex solution in which the equilibrium positions of the vortices $\xi_1, ..., \xi_n$ correspond to a minimum of the Gross-Pitaevskiy free energy. The complex-valued wave function u in dimentionless variables satisfies the evolutionary second-order nonlinear partial differential equation

$$\frac{\partial u}{\partial t} = \Delta u + \frac{1}{\varepsilon^2} u(a(\mathbf{x}) - |u|^2) + 2i(\Omega \times \mathbf{x}) \cdot \nabla u + \mu u \qquad \text{in} \quad \mathcal{D},$$

with the boundary condition $u = 0$ on $\partial\mathcal{D}$, where Ω is the angular velocity, μ is the Lagrange multiplier, and ε is the size of vortex core. The function $a(\mathbf{x})$ is $a(\mathbf{x}) = 1 - (x^2 + y^2)$ and domain D is a circle $\{a > 0 : r^2 \leq 1\}$.

The aim of this paper is to deduce a system of *ordinary differential equations* (ODEs) describing the motion of vortices. For this we construct two different asymptotic expansions for the wave function as $\varepsilon \to 0$. Matching the leading terms of these expansions yields the desired system of ODEs

$$-\frac{n_j}{2}\frac{d\xi_j^{\perp}}{dt} + \mathbf{K}_j + \frac{n_j}{1-\xi_j^2}\xi_j^{\perp} = 0, \qquad j = 1, 2, \ldots, n.$$

where $n_j = \pm 1$ is the intensity of the jth vortex, $\xi_j^{\perp} = (-y_j, x_j)$, and $\mathbf{K}_j = \mathbf{K}_j(\xi_1, \xi_2, \ldots \xi_n)$ is the gradient of the vortex solution of the auxiliary equation

$$\operatorname{div}((1 - r^2)\nabla\psi) = 0$$

given at the point of the jth vortex location. The function ψ is expressed through the Jacobi polinoms.

The equations are suitable for numerical simukations giving the results that can be easily compared with the experimental data.