On some fractional differential equations in the Hilbert space

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Abstract

Let A be a closed operator defined on a dense set in the Hilbert space H. fractional evolution equations of the form

$$\frac{d^{\alpha}u(t)}{dt^{\alpha}} = Au(t)$$

is studied in H, for a wide class of the operators, A, which in general have no resolvents, $(0 < \alpha \leq 1)$. The correct formulation of the Cauchy problem for the considered equation is studied under suitable conditions on the class of solutions. It is proved also that there exists a dense set S in H such that there exists a solution $u(t)$ of the Cauchy problem for the considered equation with the initial condition $u(0) \in S$. Applications to general partial differential equations of the form

$$\frac{\partial^{\alpha}u(x,t)}{\partial t^{\alpha}} = \sum_{|q|\leq m} a_q(x)D^q u(x,t)$$

are given without any restrictions on the characteristic form

$$\sum_{|q|=m} a_q(x)\xi^q \quad , \quad \text{where} \quad D^q = D_1^{q_1}...D_n^{q_n} \quad , \quad D_j = \frac{\partial}{\partial x_j},$$

$$\xi^q = \xi_1^{q_1}...\xi_n^{q_n} \quad , \quad |q| = q_1 + ... + q_n.$$

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