Hur hittar vi $R(A^T)$ och $N(A^T)$?

Transponera A och gör samma sak.

$$A^T = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Gauss - Jordan

$r_2 \leftarrow r_2 - r_1$

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$r_3 \leftarrow r_2 - r_3$

$r_4 \leftarrow r_4 - r_2$

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$r_5 \leftarrow r_4 - r_5$

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$r_2 \leftarrow r_2 + r_4$

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$r_1 \leftarrow r_1 + r_2$

$$\begin{array}{c} \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = T \\ \begin{array}{cccc} \beta_1 & \beta_2 & \beta_3 & \gamma_1 \end{array} \end{array}$$

$\text{rang}(T) = 3 = \text{rang}(A^T)$

Notera

- $\dim\big(R(A)\big) = \dim\big(R(A^T)\big)$

vilket alltid gäller!

Bas till $R(A^T)$ ges av $(\beta_1, \beta_2, \beta_3)$ dvs.

$$\begin{pmatrix}1\\1\\0\\0\\0\end{pmatrix}, \begin{pmatrix}-1\\0\\1\\1\\0\end{pmatrix}, \begin{pmatrix}0\\-1\\-1\\0\\1\end{pmatrix}$$

$\underline{N(A^T)}$

- $\dim(R(A^T)) + \dim(N(A^T)) = m = 4 \Rightarrow \dim(N(A)) = 4-3 = 1$

$$T = \begin{bmatrix}U\\0\end{bmatrix} \quad \text{där} \quad U = \left[\begin{array}{ccc|c}1 & 0 & 0 & -1\\0 & 1 & 0 & -1\\0 & 0 & 1 & -1\end{array}\right]$$

$$\underbrace{}_{U_\beta} \quad \underbrace{}_{U_\gamma}$$

$\boxed{x_\beta = -U_\gamma x_\gamma}$ och låt $x_\gamma^1 = e_1 = 1$

$$x_\beta^1 = -\begin{pmatrix}-1\\-1\\-1\end{pmatrix} 1 = \begin{pmatrix}1\\1\\1\end{pmatrix}$$

ger $x^1 = \begin{pmatrix}x_{\beta_1}\\x_{\beta_2}\\x_{\beta_3}\\x_{\gamma_1}\end{pmatrix} = \begin{pmatrix}1\\1\\1\\1\end{pmatrix}$ bildar bas till $N(A^T)$.

$\underline{\text{Kontroll}}$ Låt $z = x^1$

$$A^T z = \begin{pmatrix}1 & -1 & 0 & 0\\1 & 0 & -1 & 0\\0 & 1 & -1 & 0\\0 & 1 & 0 & -1\\0 & 0 & 1 & -1\end{pmatrix}\begin{pmatrix}1\\1\\1\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\\0\\0\end{pmatrix}$$ ok.