A NEW SOLVER FOR DEGREE CONSTRAINED NEVANLINNA-PICK INTERPOLATION INCLUDING DERIVATIVE CONSTRAINTS

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Abstract: This paper extends a previously proposed solver for the Nevanlinna-Pick interpolation problem with degree constraint to the problems including derivative constraints. The solver computes any real rational Nevanlinna-Pick interpolant with a degree bound by solving an optimization problem of the same type as encountered in the problem without derivative constraint. Thus, a very robust continuation method, previously devised by Nagamune for the problem without derivative constraint, can be used to solve the new optimization problem. The new solver can be used for designing general linear feedback controllers; it is particularly valuable for plants with multiple unstable poles/zeros. We demonstrate the value of the solver on a control problem by constructing a low-degree controller satisfying all specifications in a benchmark problem. In fact, the degree of our controller is only half of the degree of the controller presented in a popular modern textbook.

Keywords: Nevanlinna-Pick interpolation, derivative constraints, generalized Pick matrix, continuation method, linear robust control

1. INTRODUCTION

It is well-known that several robust control problems can be reformulated to Nevanlinna-Pick interpolation problems. Typically, we are interested in interpolants of low degree; in the controller design this corresponds to a low controller degree. Sometimes the problem involves derivative interpolation constraints, for instance in controller design with multiple poles/zeros. Thus the main purpose of this paper is to develop a solver for the *Nevanlinna-Pick interpolation problem with degree constraint* (*NPDC*), allowing for derivative constraints. We begin by formulating the classical problem of Nevanlinna-Pick interpolation including derivative constraints, but without degree constraint.

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Nevanlinna-Pick interpolation problem including derivative constraints

Suppose that two sets of complex numbers are given:

$$\mathcal{Z} := \left\{ z_j : \begin{array}{c} j = 0, 1, \ldots, n, \\ z_i \neq z_j \text{ whenever } i \neq j \end{array} \right\} \subset \mathbb{D}, \quad (1)$$

$$\mathcal{W} := \left\{ w_{j,k} : \begin{array}{c} j = 0, 1, \ldots, n, \\ k = 0, 1, \ldots, m_j - 1 \end{array} \right\}, \quad (2)$$

where $\mathbb{D}$ denotes the open unit disc $\mathbb{D} := \{z : |z| < 1\}$. The assumptions on the data are

A1 $z_0 = 0$ and $w_{0,k}$, $k = 0, 1, \ldots, m_0 - 1$, are real,
A2 $\mathcal{Z}$ and $\mathcal{W}$ are self-conjugate, that is, $(\bar{z}_j, \bar{w}_{j,k})$ is in $\mathcal{Z} \times \mathcal{W}$ whenever $(z_j, w_{j,k})$ is.

With these assumptions on $\mathcal{Z}$ and $\mathcal{W}$, the problem is to parameterize, if there exist any, all functions f that satisfy the following conditions:

C1 f is *strictly positive real*, namely, f is analytic in the closed unit disc $\overline{\mathbb{D}}$ and $\mathrm{Re}\, f(z) > 0$ for all $z \in \overline{\mathbb{D}}$,

C2 f fulfills the interpolation constraints:

$$\frac{f^{(k)}(z_j)}{k!} = w_{j,k}, \quad \begin{array}{l} j = 0, 1, \ldots, n, \\ k = 0, 1, \ldots, m_j - 1. \end{array} \tag{3}$$

This problem is a generalization of both the classical Carathéodory extension and the classical Nevanlinna-Pick interpolation problem; it reduces to Carathéodory extension (Nevanlinna-Pick interpolation) problem if we take $n = 0$ ($m_j = 1$ for all j). This generalization has already been considered, for example, in (Rosenblum and Rovnyak, 1985, Section 2.6).The necessary and sufficient condition for the existence of an interpolant in the problem above is the combination of Theorem 1 and Theorem 2 in (Georgiou, 2001).

Theorem 1.1. There exists an interpolant for the Nevanlinna-Pick interpolation problem including derivative constraints if and only if a Hermitian matrix, called the *generalized Pick matrix*,

$$P := WE + EW^H \tag{4}$$

is positive definite. Here, the matrix W is constructed with the interpolation data in the set $\mathcal{W}$ in (2) as

$$W := \begin{bmatrix} W_0 & & \\ & \ddots & \\ & & W_n \end{bmatrix}, \; W_j := \begin{bmatrix} w_{j,0} & & \\ \vdots & \ddots & \\ w_{j,m_j-1} & \cdots & w_{j,0} \end{bmatrix},$$

and the matrix E is a unique positive definite solution to the Lyapunov equation:

$$E - AEA^H = bb^T,$$

where the matrix A and the vector b are defined by the data in the set $\mathcal{Z}$ in (1) and the multiplicities $\{m_j : \; j = 0, 1, \ldots, n\}$ as follows:

$$A := \begin{bmatrix} A_0 & & & \\ & A_1 & & \\ & & \ddots & \\ & & & A_n \end{bmatrix}, \; A_j := \begin{bmatrix} z_j & & & \\ 1 & z_j & & \\ & \ddots & \ddots & \\ & & 1 & z_j \end{bmatrix},$$
$$b := \begin{bmatrix} e_1^{m_0} & e_1^{m_1} & \cdots & e_1^{m_n} \end{bmatrix}^T, \; e_1^{m_j} := \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}^T.$$

The sizes of A_j and $e_1^{m_j}$ are $m_j \times m_j$ and $m_j \times 1$, respectively.

One of the typical engineering examples where we encounter the Nevanlinna-Pick interpolation problem including derivative constraints is the H^∞ control problem with multiple unstable poles and/or zeros in a plant. Of course, we can always reduce the Nevanlinna-Pick interpolation problem with derivative constraints to the problem without them in a recursive way. However, the reduction steps are generally quite laborious, and therefore, the classical interpolation approach to such control problems has been considered to become "awkward and unwieldy" (see (Green and Limebeer, 1995, p. 18)). To the contrary, this paper will present a computational method being able to deal with the multiplicities without any difficulty.

In addition to the multiplicity issue, simplicity in the sense of real rational interpolants of low degrees is of great importance in engineering applications. The analytic interpolation theory with degree constraint that has been developed in recent years is most powerful and promising (see (Byrnes *et al.*, 2001*a*) and references therein).

The objective in this paper is to extend the applicability of an existing optimization solver for computing each real rational interpolant for *NPDC* to the cases including derivative constraints. It will turn out that the major difference from the optimization problem in *NPDC* without derivative constraint is that the objective function involves the generalized Pick matrix P in (4), instead of the plain Pick matrix. Nevertheless, the type of the optimization problem for *NPDC* with and without derivative constraints is exactly the same. Therefore, we can apply the same continuation approach as in (Enqvist, 2001; Nagamune, 2001) to solve the optimization problem.

2. NEVANLINNA-PICK INTERPOLATION WITH DEGREE CONSTRAINT INCLUDING DERIVATIVE CONSTRAINTS

The *NPDC* including derivative constraints is formulated as follows.

NPDC including derivative constraints

Given data sets $\mathcal{Z}$ and $\mathcal{W}$ in (1) and (2) that satisfy the assumptions **A1**, **A2** and

A3 the generalized Pick matrix P in (4) is positive definite,

the problem is to characterize all functions f that satisfy not only **C1** and **C2** but also

C3 f is real rational with degree at most the total number of interpolation constraints minus one, that is, $\deg f \leq m - 1, \; m := \sum_{j=0}^{n} m_j$.

Let us denote the solution set by $\mathcal{S}_{NPDC}$, i.e.,

$$\mathcal{S}_{NPDC} := \{f : f \text{ satisfies } \mathbf{C1}, \mathbf{C2} \text{ and } \mathbf{C3}\}.$$

The assumption **A3** guarantees that the set $\mathcal{S}_{NPDC}$ is nonempty, which is the reason why we set the degree bound equal the total number of interpolation constraints minus one. The most familiar element in the set $\mathcal{S}_{NPDC}$ is the so-called *central solution*.

The complete characterization of the set $\mathcal{S}_{NPDC}$ has been obtained by Byrnes and Lindquist as follows.

Theorem 2.1. (Byrnes and Lindquist, n.d.) There is a bijective map between the set of pairs of real polynomials

$$\mathcal{P}_1 := \left\{ (\alpha,\beta) : \begin{array}{c} \deg\ \alpha \le m-1, \\ \deg\ \beta \le m-1, \\ f := \beta/\alpha \in \mathcal{S}_{NPDC} \end{array} \right\},$$

and the set of real Schur polynomials

$$\mathcal{P}_2 := \left\{ \rho : \begin{array}{c} \deg \rho = m-1, \\ \rho(z) \ne 0 \ \forall |z| \ge 1 \end{array} \right\}.$$

From this theorem, we can conclude that the Schur polynomials are the characterizing factor of the set $\mathcal{S}_{NPDC}$. The natural question that may arise here is "How to actually compute the interpolant f from the preassigned real Schur polynomial ρ?" It turns out (see (Byrnes and Lindquist, n.d.)) that the computation of an interpolant f from ρ amounts to an optimization problem:

$$\begin{aligned} &\min_{q\in\mathcal{Q}_+} J_\rho(q) \\ &J_\rho(q) := \langle q+q^*, w+w^* \rangle \\ &\qquad - \left\langle \log(q+q^*), \frac{\rho\rho^*}{\tau\tau^*} \right\rangle. \end{aligned} \tag{5}$$

Here, we have introduced the following notation.

$$\mathcal{Q}_+ := \left\{ q : \begin{array}{l} \text{real rational} \\ q \in \text{span}\left\{G_{z_j,k}, \forall j, \forall k\right\} \\ q(z)+q(z^{-1}) > 0,\ \forall |z|=1 \end{array} \right\},$$

$$G_{p,m}(z) := \frac{z^m}{(1-\bar{p}z)^{m+1}},\ p\in\mathbb{D},\ m\in\{0,1,2,\dots\},$$

w : a square integrable function with vanishing negative Fourier coefficients satisfying (3),

$$\tau(z) := \prod_{j=1}^{n} (1-\bar{z}_j z)^{m_j},$$

$$\langle f,g \rangle := \frac{1}{2\pi}\int_{-\pi}^{\pi} f(e^{i\theta}) g^*(e^{i\theta}) d\theta,$$

where $\binom{n}{k}$ means the binomial coefficients. The optimization problem is convex, that is, the domain $\mathcal{Q}_+$ is a convex region and the objective function J_ρ is a strictly convex function. Furthermore, as a consequence of Proposition 3.1 below, the solution of the problem (5) is independent of the choice of w. After obtaining the minimizer q in $\mathcal{Q}_+$, the real polynomials α and β of degree not greater than $m-1$ can be calculated respectively by spectral factorization:

$$q(z)+q(z^{-1}) = a(z)a(z^{-1}) := \frac{\alpha(z)\alpha^*(z)}{\tau(z)\tau^*(z)}, \tag{6}$$
$$\alpha(z) = \alpha_0 + \alpha_1 z + \cdots + \alpha_{m-1} z^{m-1},$$

and by solving a system of linear equations:

$$\alpha(z)\beta(z^{-1}) + \alpha(z^{-1})\beta(z) = \rho(z)\rho(z^{-1}). \tag{7}$$

The first breakthrough about the convex optimization (5) was done in (Byrnes *et al.*, 1998) for the Carathéodory (covariance) extension problem with degree constraint, followed by the work for the Nevanlinna-Pick interpolation problem with degree constraint in (Byrnes *et al.*, 2000; Byrnes *et al.*, 2001*b*). However, these solvers were not quite robust numerically. Especially, the solvers cannot obtain interpolants with poles in close vicinity of the unit circle accurately. The solvers have been modified with a continuation homotopy method by Enqvist for Carathéodory extension in (Enqvist, 2001), followed by the second author for Nevanlinna-Pick interpolation in (Nagamune, 2001).

3. A NEW OPTIMIZATION PROBLEM FOR *NPDC* INCLUDING DERIVATIVE CONSTRAINTS

Here we will show that the optimization problem in (5) is equivalent to another optimization problem with respect to a real vector

$$\boldsymbol{\alpha} := [\alpha_0, \alpha_1, \cdots, \alpha_{m-1}]^T \in \mathbb{R}^m, \tag{8}$$

which consists of coefficients of a polynomial α in (6). When we substitute (6) into the objective function $J_\rho(q)$ in (5), we obtain a function of $\boldsymbol{\alpha}$ as

$$\hat{g}_\rho(\boldsymbol{\alpha}) := \langle aa^*, w+w^* \rangle - \left\langle \log aa^*, \frac{\rho\rho^*}{\tau\tau^*} \right\rangle. \tag{9}$$

The first and second terms of $\hat{g}_\rho$ will be examined separately.

The first term in (9) can be written as a quadratic form of $\boldsymbol{\alpha}$ containing the generalized Pick matrix. To this end, the following proposition represents the first term of $\hat{g}_\rho$ as a function of a complex vector $\boldsymbol{\gamma} \in \mathbb{C}^m$, where

$$\boldsymbol{\gamma} := \begin{bmatrix} \boldsymbol{\gamma}_0 \\ \boldsymbol{\gamma}_1 \\ \vdots \\ \boldsymbol{\gamma}_n \end{bmatrix},\ \boldsymbol{\gamma}_k = \begin{bmatrix} \gamma_{k,0} \\ \gamma_{k,1} \\ \vdots \\ \gamma_{k,m_k-1} \end{bmatrix},$$

and the scalars $\gamma_{k,j}$ are defined by

$$a(z) =: \sum_{k=0}^{n} \sum_{j=0}^{m_k-1} \gamma_{k,j} G_{z_k,j}(z).$$

Proposition 3.1. For any w defined as in (5), the following holds:

$$\langle aa^*, w + w^* \rangle = \boldsymbol{\gamma}^H P \boldsymbol{\gamma}, \tag{10}$$

where the matrix P is the generalized Pick matrix defined in (4), and the vector $\boldsymbol{\gamma}$ is defined as above. The proof is omitted for the sake of brevity.

Therefore the first term of $\hat{g}_\rho$ in (9) is expressed as a function of $\boldsymbol{\gamma}$ in (10). Below, we clarify the relation between vectors $\boldsymbol{\gamma}$ and $\boldsymbol{\alpha}$ in (8) in order to transform (10) into a function of $\boldsymbol{\alpha}$. By the similar arguments to (Nagamune, 2001, Lemma 3.1), we can derive the relation between $\boldsymbol{\gamma}$ and $\boldsymbol{\alpha}$ as

$$\boldsymbol{\alpha} = L_m V \boldsymbol{\gamma}. \tag{11}$$

Here, the $m \times m$ nonsingular matrices L_m and V are written as

$$L_m := \begin{bmatrix} 1 & & & \\ \tau_1 & 1 & & \\ \vdots & \ddots & \ddots & \\ \tau_{m-1} & \cdots & \tau_1 & 1 \end{bmatrix},$$
$$V := [V_0 \; V_1 \; \cdots \; V_n],$$

where the $m \times m_k$ block matrices V_k are defined by

$$V_k := \begin{bmatrix} 1 & & & \\ \bar{z}_k & 1 & & \\ \bar{z}_k^2 & \binom{2}{1}\bar{z}_k & \ddots & \\ \vdots & \vdots & \ddots & 1 \\ \vdots & \vdots & & \binom{m_k}{1}\bar{z}_k \\ \vdots & \vdots & & \vdots \\ \bar{z}_k^{m-1} & \binom{m-1}{m-2}\bar{z}_k^{m-2} & & \binom{m-1}{m-m_k}\bar{z}_k^{m-m_k} \end{bmatrix}.$$

Due to Proposition 3.1 and the relation (11), the first term of $\hat{g}_\rho$ amounts to a representation with the vector $\boldsymbol{\alpha}$ as $\langle aa^*, w + w^* \rangle = \boldsymbol{\alpha}^T K \boldsymbol{\alpha}$, where the positive definite matrix K is defined by $K := L_m^{-T} V^{-H} P V^{-1} L_m^{-1}$.

The second term of $\hat{g}_\rho$ in (9) becomes

$$\left\langle \log aa^*, \frac{\rho\rho^*}{\tau\tau^*} \right\rangle = \left\langle 2\log|\alpha| - 2\log|\tau|, \frac{\rho\rho^*}{\tau\tau^*} \right\rangle.$$

Since α is a real polynomial, the following holds:

$$\left\langle \log|\alpha|, \frac{\rho\rho^*}{\tau\tau^*} \right\rangle = \left\langle \log\alpha, \frac{\rho\rho^*}{\tau\tau^*} \right\rangle.$$

To summarize, the function $\hat{g}_\rho$ in (9) has become

$$\hat{g}_\rho(\boldsymbol{\alpha}) = \boldsymbol{\alpha}^T K \boldsymbol{\alpha} - 2\left\langle \log\alpha - \log\tau, \frac{\rho\rho^*}{\tau\tau^*} \right\rangle,$$

where $\boldsymbol{\alpha}$ should be in the Schur stability region:

$$\mathcal{S}_m := \left\{ \boldsymbol{\alpha} \in \mathbb{R}^m : \; \alpha(z) \neq 0, \forall z \in \overline{\mathbb{D}}, \; \alpha_0 > 0 \right\},$$

where $\overline{\mathbb{D}}$ denotes the closure of $\mathbb{D}$. Since the last term does not include the polynomial α, it is nothing to do with the optimization of $\hat{g}_\rho$. Thus, the new optimization problem equivalent to (5) is

$$\min_{\boldsymbol{\alpha} \in \mathcal{S}_m} g_\rho(\boldsymbol{\alpha}),$$
$$g_\rho(\boldsymbol{\alpha}) := \boldsymbol{\alpha}^T K \boldsymbol{\alpha} - 2\left\langle \log\alpha, \frac{\rho\rho^*}{\tau\tau^*} \right\rangle.$$

Since K is positive definite, this is exactly the same kind of problem that has been dealt with in (Enqvist, 2001; Nagamune, 2001). More precisely, the function g_ρ has the following properties (see (Nagamune, 2001)):

- unique stationary point in the open region $\mathcal{S}_m$, and
- locally strictly convex around the unique stationary point.

Therefore, we can use the same technique as in (Nagamune, 2001), that is, the continuation homotopy method, to solve this optimization problem. The details of the optimization has been presented in (Nagamune, 2001). With the optimal $\boldsymbol{\alpha}$, the interpolant f is calculated by solving (7) with respect to β.

4. A CONTROL EXAMPLE

We take a controller design example which amounts to *NPDC* including derivative constraints from a book written by Doyle, Francis & Tannenbaum (DFT), (Doyle *et al.*, 1992, Section 10.3 & 12.4). A plant is given as

$$P(s) = \frac{-6.4750s^2 + 4.0302s + 175.7700}{s(5s^3 + 3.5682s^2 + 139.5021s + 0.0929)},$$

that is, it has one unstable pole at the origin and unstable zeros at infinity (multiplicity 2) and 5.5308. For this plant, our goal is to design a strictly proper controller, $C(s)$, in the feedback structure of Figure 1, fulfilling the specifications:

- Internal stability of the closed-loop system.
- Settling time of at most 8 seconds.
- Overshoot of at most 10 %.

- Control signal $u(t)$ of at most magnitude 0.5 for a step reference signal $r(t)$.

For the controller design, we define a sensitivity function

$$S(s) := \frac{1}{1 + P(s)C(s)},$$

which is the transfer function from the reference signal r to the error e. Our approach is to find an appropriate S so that the closed-loop system fulfills all the specifications. Afterwards, we calculate the corresponding controller C.

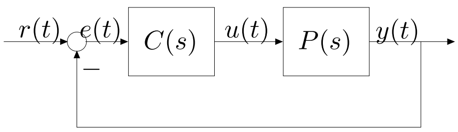

Figure 1. The closed-loop system

For internal stability, the sensitivity function S must be in the set of real rational stable functions, denoted by RH^∞, and we must not have any pole-zero cancellation of PC in the right half-plane. The second condition translates into the interpolation conditions of S as

$$S(0) = 0,\ S(\infty) = 1,\ S(5.5308) = 1,$$
$$S'(\infty) = S''(\infty) = 0,$$

where the last constraint makes the controller strictly proper and is easily included in our approach.

In the frequency domain, the given time domain specifications correspond to the sensitivity function (see (Doyle *et al.*, 1992, p. 181)):

$$S_{ideal}(s) := \frac{s(s + 1.2)}{s^2 + 1.2s + 1}.$$

Therefore, we will aim at getting a similar frequency domain characteristics of our interpolant as that of S_{ideal}. In our design procedure, we need the upper bound of the H^∞ norm. We choose this value, γ, comparable to that of S_{ideal}. In addition to the above requirements from (Doyle *et al.*, 1992) we try to obtain a controller of low degree.

We will perform our design in the following steps:

1. Identify the interpolation data set.
2. Transform the interpolation problem to our formulation.
3. Choose design parameters.
4. Solve the problem with the proposed solver and transform the interpolant back.
5. Determine the corresponding controller and evaluate the performance.
6. Iterate steps 2–5 until the desired performance is achieved.

As identified above, the set of interpolants is

$$\mathcal{S}_{NPDC} := \left\{ S : \begin{array}{c} S \in RH^\infty,\ S(0) = 0, \\ S(5.5308) = 1\ S(\infty) = 1, \\ S'(\infty) = S''(\infty) = 0, \\ \|S\|_\infty \le \gamma,\ \deg S \le 4 \end{array} \right\}.$$

The bound of the degree of S is a consequence of having the total number of interpolation constraints as five. That corresponds to a controller degree of at most $\deg P - 1 = 4$ (see (Nagamune, 2000, Proposition 4.1) and (Nagamune and Lindquist, 2001, Proposition 2.1)). This also implies that we get four spectral zeros as design parameters.

We will need to transform both the domain and the range of the sensitivity function to get it on our form. We use all the transformations in Table 1.

	Transformation	Original Region	New Region
T1	$w = \frac{-z+1}{z+1}$	$Re\{z\} \ge 0$	$\lvert w\rvert \le 1$
T2	$w = \kappa z$	$\lvert z\rvert \le \frac{1}{\kappa}$	$\lvert w\rvert \le 1$
T3	$w = \frac{z - z_0}{-\bar{z}_0 z + 1}$	$\lvert z\rvert \le 1$	$\lvert w\rvert \le 1$
		$z_0 \in \mathbb{D}$	0

Table 1. Useful bilinear transformations

We choose the design parameters based on the follow reasoning: we need to choose an H^∞ norm $\gamma = 1.8$ causing the peak of the sensitivity function to be slightly lower than that of the DFT-design. Putting the first spectral zeros in $s = \pm 1.7i$ corresponding to the frequency for which we want the peak of the magnitude of the sensitivity. The remaining two spectral zeros we place in $s = 7$ and $s = \infty$, which brings the magnitude of the control signal down. However, it should be noted that the effect of spectral zeros away from the imaginary axis is quite unclear. The resulting controller is

$$C_{NPDC}(s) = \frac{12.63s^3 + 9.016s^2 + 352.5s + 0.2347}{s^4 + 20.15s^3 + 139.2s^2 + 448.8s + 650.7} \quad (13)$$

We compare the closed-loop performance of the controller (13) with that of the controller in (Doyle *et al.*, 1992, Section 12.4) shown in (12) at the bottom of the page.

$$C_{DFT}(s) = \frac{1.424s^7 + 907.6s^6 + 31410s^5 + 11170s^4 + 907300s^3 + 1961000s^2 + 1306s + 0.01406}{s^8 + 1013s^7 + 13260s^6 + 112900s^5 + 632600s^4 + 2348000s^3 + 4940000s^2 + 3440000s + 3435} \quad (12)$$

The performance of the different designs, both in the frequency and time domain, is summarized in Table 2. The behavior can also be seen in the step response in Figure 2.

We have clearly found an at least as good design but of the half degree compared with the DFT-design.

	DFT	**NPDC**
Controller degree	8	4
Peak Gain	1.56	1.55
Bandwidth (Hz)	0.48	0.52
Rise Time (sec)	1.55	1.46
Overshoot	1.11	1.02
Settling Time (sec)	5.41	2.49
Max $\|u\|$	0.48	0.48

Table 2. The time and frequency domain performance.

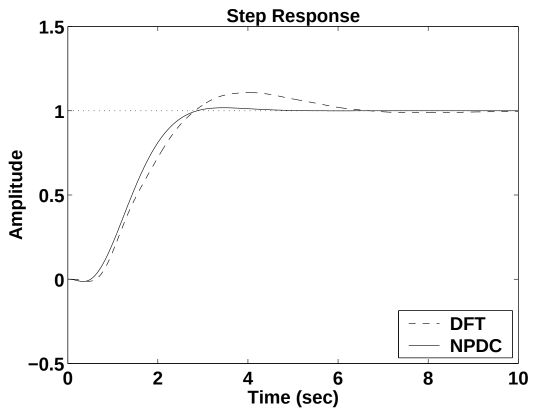

Figure 2. The step response $y(t)$ for the closed loop systems with the DFT-design and our design.

5. CONCLUSIONS

In this paper, we have shown that the Nevanlinna-Pick interpolation problem with degree constraint including derivative constraints can be treated in the same framework as the problem without derivative constraint. We can obtain each interpolant by solving the same kind of optimization problem as the one that appears in the plain Nevanlinna-Pick interpolation with degree constraint. The major difference is in the construction of a positive definite matrix in the objective function, which contains the generalized Pick matrix. We have written a MATLAB code based on a continuation method. We have demonstrated that the software is quite convenient for a sensitivity shaping in control when a plant has multiple unstable poles/zeros, yielding derivative constraints, as well as when we want to design a strictly proper controller of low degree.

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