



## Computer Exercise 1 Mathematical Systems Theory, 5B1742

Fall 2002

Satisfactory completion of this computer exercise gives two (2) bonus credits for this year's finals. The examination of the computer exercises is *oral and written*. Firstly the results should be presented in a written report (use a word processor) and secondly each student should be prepared to answer question about the exercise when it is handed back. Hand in the report at the latest during the exercise session on October 2 and pick it up after agreement with Anders Blomqvist. Do attach your commented MATLAB code to your report! Cooperation in groups of not more than two students is allowed.

The necessary MATLAB m-files can be found at [www.math.kth.se/~andersb/teaching.html](http://www.math.kth.se/~andersb/teaching.html). The intention of this exercise is to exemplify how easy it is to apply the theory of the course using a computer. We will use the "Control System Toolbox" in MATLAB, which will save us lots of programming. Names written with bold font are command names in MATLAB. Write **help control** to get a list of available functions in the "Control System Toolbox" or use the help browser.

### 1. REALIZATION THEORY, STABILITY AND POLE PLACEMENT.

The following transfer matrix is given:

$$R(s) = \begin{bmatrix} \frac{1}{s^2} & \frac{1}{s-1} \\ \frac{1}{s} & 0 \end{bmatrix}.$$

In the file "realization.m" the matrices  $A, B, C, D$  that correspond to the trivial realization of the transfer matrix are given (trivial in the sense of being a realization of each component of the transfer matrix)

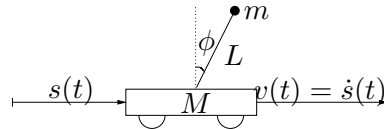
- (1) Verify using **ss2tf** that  $(A, B, C, D)$  indeed is a realization of  $R(s)$ . (Observe that Matlab doesn't simplify the generated polynomials.)
- (2) Is the realization reachable, observable or minimal? (Use **ctrb**, **obsv**.)
- (3) Determine a minimal realization using **minreal**.  
How do you think **minreal** is implemented? (Try **type minreal**.)
- (4) Verify that the minimal realization is both reachable and observable and that it still is a realization of  $R(s)$ ? (Use **ss2tf**.)
- (5) Determine if the system is stable,
  - (a) first by using the Lyapunov method, (Use **lyap**)
  - (b) secondly by determining the location of the poles. (Use e.g. **poly**, **roots**)
- (6) Move the poles to the left half plane.  
Use **place**: note the Matlab convention " $A - BK$ ".
- (7) Is it possible to place all the poles in  $\{-1\}$  using **place**? Why doesn't this contradict the pole-placement theorem?

## 2. THE INVERTED PENDULUM

In the file “pendulum.m” the matrices  $A, B, C, D$  that correspond to the linearized realization of the pendulum with the states

$$x = [\phi \quad \dot{\phi} \quad s \quad \dot{s}]'$$

are given.



- (1) Is the realization reachable, observable or minimal ?
- (2) Determine whether the system is stable by determining the location of the poles. (Use e.g. **poly**, **roots**)
- (3) Design three different feedbacks, all with closed-loop poles in the left half-plane.
  - (a) All poles close to the origin.
  - (b) All poles with small imaginary parts but large real parts.
  - (c) All poles with large imaginary parts but small real parts.
- (4) Now study the closed loop system by
  - (a) determining the impulse response, equivalent to a chock on the cart.  
Use **impulse** , and the time vector **t=0:0.05:10** .
  - (b) determining the step response, equivalent to moving the cart sideways.  
Use **step** , and the time vector **t=0:0.1:40** .

Compare the feedback design from above and explain the differences.
- (5) Now let the system be described by  $\dot{x} = (A+BK)x + Sw$ , where  $S = [0 \ 1 \ 0 \ 0]'$ . We let  $w$  describe disturbances on the pendulum, caused by wind and annoying flies. Make a random vector of normalized white noise by **w=randn(1,length(t))/10** and simulate the system using **lsim** for the different feedback designs.

*Good Luck! Don't hesitate to ask (by email or phone) if anything is unclear.*

APPENDIX: POLE PLACEMENT IN THE MULTI-INPUT CASE.

This section is for reference only. It does not contain any exercises. This pole placement algorithm is based on the proof of the Pole-Assignment Theorem of the lecture notes.

Main Idea : Transform to the single input case.

Algorithm.

Given is

$$U = [ B \quad AB \quad \dots \quad A^{n-1}B ] \quad \text{of rank } n.$$

and let  $B$  have the columns  $b_1, \dots, b_m$ .

(I): Determine the matrix  $M$  via

$$M = [ b_1 \quad Ab_1 \quad \dots \quad A^{r_1-1}b_1, \quad b_2 \quad Ab_2 \quad \dots \quad A^{r_2-1}b_2, \quad b_3 \quad \dots ].$$

and  $r_i$  is the largest number such that the rank of  $M$  increases (the  $r_i$  are called *controllability indices* and are invariants of the system under feedback).

The  $n \times n$ -matrix  $M$  is now invertible.

(II): Define the matrix  $N$  as  $N = [ \eta_1 \quad \eta_2 \quad \dots \quad \eta_n ]$ , where

$$\eta_k = \begin{cases} e_{i+1} & \text{if } k = r_i \text{ for some } i, \\ 0 & \text{if } k \neq r_i. \end{cases}$$

and  $e_i$  are the unit vectors in  $\mathbb{R}^n$ .

(III): Let  $K_1 = NM^{-1}$ .

(IV): Now  $[A + BK_1, b_1]$  is reachable, so with  $u = (K_1 + K)x$  and

$$K = \begin{bmatrix} - & k & - \\ & 0 & \end{bmatrix},$$

we get  $\dot{x} = (A + BK_1)x + b_1 kx$  which is a single input poleplacement problem in  $k$ .

For problem 1.7 on the computer exercise, this gives

```
b1=Bm(:,1);b2=Bm(:,2);
M=[b1 Am*b1 b2]
N=[[0;0] [0;1] [0;0]]
K1=N*inv(M)
AA=Am+Bm*K1
```

Now we have a single input problem for **AA**, **b1**, which can be solved by, (or by using **place**.)

```
G=ctrb(AA,b1)
t=(G'\[0 0 1]')'
T=[t ; t*AA ; t*AA^2]
p=poly(AA)
g=flipplr(p(1,2:4)-[3 3 1])
k=g*T
```

```
K2=[k;0 0 0]
poly(Am+Bm*K1+Bm*K2) % to make sure ...
```