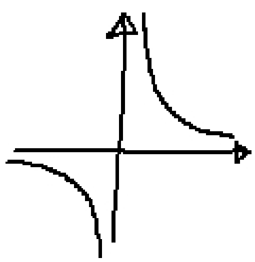
$$\lim_{s \to 0} (1+s)^{\frac{1}{s}} = e$$



$$\Rightarrow \lim_{s \to 0^+} (1+s)^{\frac{1}{s}} = e$$

men vi kan också skriva om det

Sätt $t = \frac{1}{s}$ $\quad s \to 0^+ \Rightarrow t \to \infty$

$$e = \lim_{s \to 0^+} (1+s)^{\frac{1}{s}} = \lim_{t \to \infty} \left(1 + \frac{1}{t}\right)^t$$

$$e = \lim_{s \to 0^-} (1+s)^{\frac{1}{s}} = \lim_{t \to -\infty} \left(1 + \frac{1}{t}\right)^t$$

Ex typ 304 b, g:

$$\lim_{x\to 0} \frac{\ln(1+2x)}{x} = \lim_{x\to 0} \ln\left((1+2x)^{\frac{1}{x}}\right) =$$

$$= \lim_{x\to 0} \ln\left((1+2x)^{\frac{2}{2x}}\right) = \left[\begin{array}{c} t = 2x \\ x\to 0 \Rightarrow t\to 0 \end{array}\right] =$$

$$= \lim_{t\to 0} \ln\left(\left((1+t)^{\frac{1}{t}}\right)^2\right) = \lim_{t\to 0} 2\ln\left((1+t)^{\frac{1}{t}}\right) =$$

$$\left[(a^b)^c = a^{bc} \right] = 2 \lim_{t\to 0} \ln\left((1+t)^{\frac{1}{t}}\right) =$$

$$= (\ln \text{ kontinuerlig}) = 2\ln\left(\lim_{t\to 0} (1+t)^{\frac{1}{t}}\right) =$$

$$\left[\lim_{x\to a} \ln x = \ln a = \ln\left(\lim_{x\to a} x\right) \right] = 2\ln e = 2.$$

Ex 401z: $D\,x^x$

$$D\,x^x = D\,e^{x\ln x} = e^{x\ln x}\,D(x\ln x) =$$

$e^{x\ln x} = (e^{\ln x})^x$

$$= e^{x\ln x}\left(Dx\cdot \ln x + x\,D\ln x\right) =$$

$$= e^{x\ln x}\left(\ln x + x\frac{1}{x}\right) = x^x(\ln x + 1)$$

$$D\,x^{x^2} = D\,e^{x^2\ln x} = e^{x^2\ln x}\,D(x^2\ln x) =$$

$$= x^{x^2}\left(2x\ln x + x^2\frac{1}{x}\right) = x^{x^2}(2x\ln x + x)$$

$$= x^{x^2+1}(2\ln x + 1)$$

Ex $D\left((1+x)^3(1-x)^4\right)$

401 c)

$$D\left(\underset{f(x)}{(1+x)^3}\cdot\underset{g(x)}{(1-x)^4}\right) = D\left((1+x)^3\right)\cdot(1-x)^4 +$$

$$+ (1+x)^3\, D\left((1-x)^4\right) = 3(1+x)^2(1-x)^4 +$$

$$+ (1+x)^3\, 4(1-x)^3(-1) = (1+x)^2(1-x)^3\left(3(1-x)-4(1+x)\right)$$

$$= (1+x)^2(1-x)^3(3-4-3x-4x) =$$

$$= (1+x)^2(1-x)^3(-1-7x)$$

Differentialer, differensformeln

$$F'(x) = \lim_{\Delta x \to 0} \frac{F(x+\Delta x) - F(x)}{\Delta x}$$

$$\lim_{\Delta x \to 0} \underbrace{\left(\frac{F(x+\Delta x) - F(x)}{\Delta x} - F'(x) \right)}_{\rho(\Delta x)} = 0$$

$$\rho(\Delta x) = \frac{F(x+\Delta x) - F(x)}{\Delta x} - F'(x)$$

$$F(x+\Delta x) - F(x) = F'(x)\Delta x + \rho(\Delta x)\Delta x$$

$\rho(\Delta x) \to 0$ när $\Delta x \to 0$.

Antag att det finns en konstant A, så att

$$(*) \qquad F(x+\Delta x) - F(x) = A\Delta x + \rho(\Delta x)\Delta x$$

där ρ är en funktion sådan att

$$\lim_{\Delta x \to 0} \rho(\Delta x) = 0.$$

$$\lim_{\Delta x \to 0} \frac{F(x+\Delta x) - F(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{A\Delta x + \rho(\Delta x)\Delta x}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} A + \rho(\Delta x) = A$$

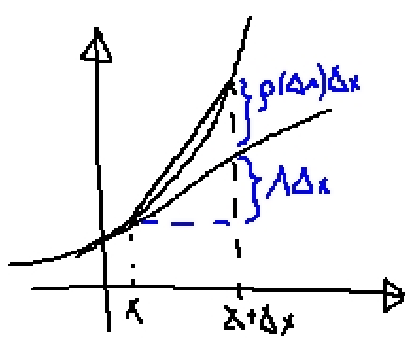
Alltså: $(*) \Rightarrow F$ deriverbar i x och $A = F'(x)$

Sats: F deriverbar i punkten x

$$\Longleftrightarrow$$

finns en konstant A och en funktion $\rho(x)$, $\rho(x) \to 0$ när $x \to 0$, så att

$$F(x+\Delta x) - F(x) = A\Delta x + \rho(\Delta x)\Delta x$$



$F'(x)\Delta x = dF$ kallas differentialen för F i punkten x.

Ex: $\underset{x}{(1} + \underset{\Delta x}{0{,}1})^2 = 1{,}21 \qquad F(x) = x^2$

$$(1+0{,}1)^2 - 1 = 0{,}21$$

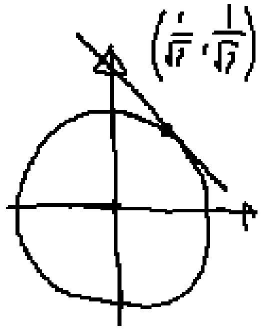
$$F'(x) = 2x$$

$$F'(1)\,0{,}1 = 2 \cdot 0{,}1 = 0{,}2$$

$y' = -\dfrac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = -1$

Implicit Derivering.

$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

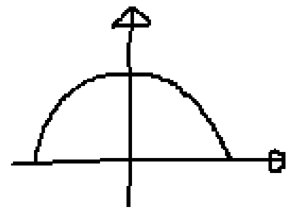


$x^2 + y^2 = 1 \Rightarrow$ Deriverar båda sidor map x

$$\Rightarrow 2x + 2yy' = 0$$

$$\Rightarrow y' = -\frac{x}{y}$$

$$y = \sqrt{1 - x^2}$$



$$y' = \frac{1}{2} \frac{1}{\sqrt{1 - x^2}} (-2x)$$

$$= \frac{-x}{\sqrt{1 - x^2}} = -\frac{x}{y}$$

Ex: $y^3 + xy = x^2$

$$\boxed{D\, f(x)^3 = 3 f(x)^2 \cdot f'(x)}$$

Implicit derivering ger

$$3y^2 y' + y + xy' = 2x$$

$$\Rightarrow \quad y'(3y^2 + x) = 2x - y$$

$$y' = \frac{2x - y}{3y^2 + x}$$

Logaritmisk derivering

Ex: 401z: $y = x^x = e^{x\ln x}$

$\Leftrightarrow \ln y = x \ln x$

Vi deriverar båda sidor

$$\frac{1}{y} y' = 1 \cdot \ln x + \frac{x}{x} = \ln x + 1$$

$$\Rightarrow y' = y(\ln x + 1) = x^x(\ln x + 1)$$

Ex: $y = \tan x = \frac{\sin x}{\cos x}$

$$\ln y = \ln\left(\frac{\sin x}{\cos x}\right) = \ln(\sin x) - \ln(\cos x)$$

Vi deriverar båda sidor

$$\frac{y'}{y} = \frac{\cos x}{\sin x} - \frac{-\sin x}{\cos x} \Rightarrow$$

$$y' = y\left(\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}\right) = \frac{\sin x}{\cos x}\left(\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}\right) = 1 + \tan^2 x$$

Övn: beräkna $D\left((1+x)^5(1-x)^4\right)$
med hjälp av logaritmisk
derivering

(typ 315) Ex: Låt $f(x) = \frac{\sin x}{x}$ $x \neq 0$.
Uppgift: Definiera $f(x)$ i $x=0$ så att
f blir kontinuerlig.

Lösn: Vi vet att $\lim_{x \to 0} \frac{\sin x}{x} = 1$

För att f ska bli kontinuerlig
måste $f(0) = 1$.