Grön

1. Sätt $f(0) = \lim_{x \to 0} (1+x^2)^{\frac{1}{x^2}}$

$$\lim_{x \to 0} (1+x^2)^{\frac{1}{x^2}} = \begin{bmatrix} t = x^2 \\ x \to 0 \\ t \to 0^+ \end{bmatrix} = \lim_{t \to 0^+} (1+t)^{\frac{1}{t}} =$$

$= e$. Alltså $f(0) = e$

2. $$D^3(e^x x^3) = \sum_{k=0}^{3} \binom{3}{k} D^k e^x D^{3-k} x^3 =$$

$$= e^x(6 + 3 \cdot 6x + 3 \cdot 3x^2 + x^3)$$

Gul

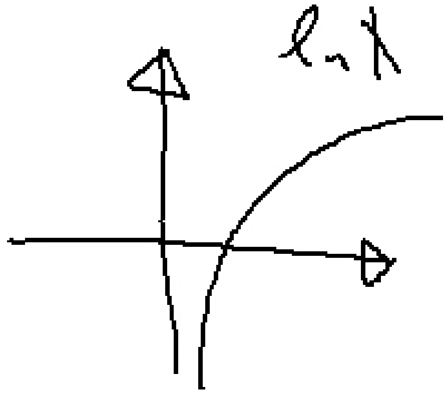
1. $$\lim_{x \to 0} \arctan \ln|x| =$$

$$\begin{bmatrix} t = \ln|x| \\ x \to 0 \\ t \to -\infty \end{bmatrix} = \lim_{t \to -\infty} \arctan t = -\frac{\pi}{2}.$$

2. $$D\, x^{\sin x} = D(e^{\sin x \ln x})$$

$$= e^{\sin x \ln x} D(\sin x \ln x) =$$

$$= x^{\sin x} \left(\cos x \ln x + \frac{\sin x}{x} \right)$$



Rosa

$$\lim_{x\to 0}\frac{e^x-1}{x} = De^x\Big|_{x=0} = 1$$

$$De^x\Big|_{x=0} = \lim_{\Delta x\to 0}\frac{e^{\Delta x}-e^0}{\Delta x} = \lim_{\Delta x\to 0}\frac{e^{\Delta x}-1}{\Delta x}$$

$$f(0) = 1.$$

$$-12x(1-x)^3(1+2x)$$

2. $$D\left((1-x)^4(1+2x)^2\right) =$$

$$-4(1-x)^3(1+2x)^2 + (1-x)^4\,2(1+2x)\cdot 2 =$$

$$= (1-x)^3(1+2x)\left(-4(1+2x)+4(1-x)\right) =$$

$$\lim_{x\to 0}\frac{e^x-1}{x} = \lim_{x\to 0}\frac{1+x+O(x^2)-1}{x} =$$

$$= \lim_{x\to 0}\frac{x+O(x^2)}{x} = \lim_{x\to 0} 1+O(x) = 1$$

Ex: $$\lim_{x\to 0}\frac{1-\cos x}{x^2} = \lim_{x\to 0}\frac{1-\left(1-\frac{x^2}{2}+O(x^4)\right)}{x^2}$$

MacLaurinutveckling av $\cos x$

$$\cos x = 1-\frac{x^2}{2}+\frac{x^4}{4!}-\frac{x^6}{6!}+O(x^8)$$

$$= \lim_{x\to 0}\frac{\cancel{\frac{x^2}{2}}+O(x^{\cancel{4}\,2})}{\cancel{x^2}} = \frac{1}{2}$$

$$\lim_{x\to 0} \frac{1-\cos^2 x}{x^2} = \lim_{x\to 0} \frac{1-\left(1-\frac{x^2}{2}+O(x^4)\right)^2}{x^2}$$

$$= \lim_{x\to 0} \frac{\cancel{1}-(\cancel{1}-x^2+O(x^4))}{x^2} = 1$$

$$\lim_{x\to 0} \frac{\sin^2 x}{x^2} = \lim_{x\to 0} \frac{(x+O(x^3))^2}{x^2} =$$

$$= \lim_{x\to 0} \frac{x^2+O(x^3)}{x^2} = 1$$

$$\frac{x^2}{2}\cdot O(x^4) = O(x^6)$$

$$(a+b+c)^2 = a^2+b^2+c^2+2ab+2ac+2bc$$

$$\lim_{x\to\infty} x^2\left(\frac{1}{1+x}-\frac{1}{x}\right) = \left[\begin{array}{l} t = \frac{1}{x} \\ x \to \infty \\ t \to 0^+ \end{array}\right] =$$

$$= \lim_{t\to 0^+} \frac{1}{t^2}\left(\frac{1}{1+\frac{1}{t}} - t\right) =$$

$$= \lim_{t\to 0^+} \frac{1}{t^2}\left(\frac{t}{1+t} - t\right) = \lim_{t\to 0^+} \frac{1}{t}\left(\frac{1}{1+t} - 1\right) =$$

$$= \lim_{t\to 0^+} \frac{1}{t}\left(1-t+O(t^2) - 1\right) = \lim_{t\to 0^+} \frac{-t+O(t^2)}{t} =$$

$$= -1$$

$$\text{Ex: } \lim_{x\to\infty} \sqrt{x^2+2x} - x = \left[\begin{matrix} x = \frac{1}{t} \\ x\to\infty \\ t\to 0^+\end{matrix}\right]$$

$$= \lim_{t\to0^+} \sqrt{\frac{1}{t^2}+\frac{2}{t}} - \frac{1}{t} = \lim_{t\to0^+}\frac{\sqrt{1+2t}-1}{t}$$

$$= \lim_{t\to0^+}\frac{1+\frac{1}{2}2t + O(t^2) - 1}{t} =$$

$$\left[(1+x)^\alpha = 1+\alpha x + O(x^2)\right]$$

$$= \lim_{t\to0^+}\frac{t+O(t^2)}{t} = 1$$

$$\lim_{x\to0}\frac{\ln(1+3x)}{x(x+2)} = \lim_{x\to0}\frac{3x - \frac{3x^2}{2}+O(x^3)}{x^2+2x} =$$

$$\left[\begin{aligned} \ln(1+x) &= \ln 1 + D\ln(1+x)\Big|_{x=0} x + D^2\ln(1+x)\Big|_{x=0}\frac{x^2}{2} + O(x^3) \\ &= 0 + \frac{1}{1+x}\Big|_{x=0} x + \left(-\frac{1}{(1+x)^2}\right)\Big|_{x=0}\frac{x^2}{2} + O(x^3) \\ &= x - \frac{x^2}{2} + O(x^3) \end{aligned}\right.$$

$$= \lim_{x\to0}\frac{3-\frac{3x}{2}+O(x^2)}{2+x} = \frac{3}{2}$$

421 c) Bestäm största och minsta värdet till

$$f(x) = \frac{2x+3}{x^2+1} + 3\arctan x$$

om de finns.

Lösn: funktionen är definierad överallt, kontinuerlig och deriverbar.

$$f'(x) = \frac{2(1+x^2)-(2x+3)2x}{(1+x^2)^2} + \frac{3}{1+x^2} =$$

$$= \frac{x^2-6x+5}{(1+x^2)^2}$$

$$f'(x)=0 \iff x^2-6x+5=0$$

$$\Rightarrow x=1 \text{ eller } x=5.$$

$$f(1) = \frac{5}{2} + 3\frac{\pi}{4}, \quad f(5) = \frac{1}{2} + 3\arctan 5$$

		1		5		
$\frac{(x^2-6x+5)}{(1+x^2)^2} = f'(x)$	$+++$	0	$---$	0	$+++$	$(x-1)(x-5)$
f	$\nearrow\nearrow$	lok Max	$\searrow\searrow$	lok Min	$\nearrow\nearrow$	

$$\lim_{x\to\infty} \underbrace{\frac{2x+3}{x^2+1}}_{\to 0} + 3\arctan x = \frac{3\pi}{2}$$

$$\lim_{x\to-\infty} \underbrace{\frac{2x+3}{x^2+1}}_{\to 0} + 3\arctan x = -\frac{3\pi}{2}$$

Vi ska jämföra $\frac{3\pi}{2}$ med $f(1)$, dvs $\frac{3\pi}{2}$ och $\frac{5}{2} + \frac{3\pi}{4}$. $\frac{5}{2} + \frac{3\pi}{4} > \frac{3\pi}{2}$.

antas aldrig!

$\Rightarrow$ finns inget minsta värde

$-\frac{3\pi}{2} < \frac{1}{2} + 3\arctan 5$.

Så $f(5)$ ej minsta värdet

$(3\pi < 10 \Rightarrow \frac{3\pi}{4} < \frac{5}{2})$