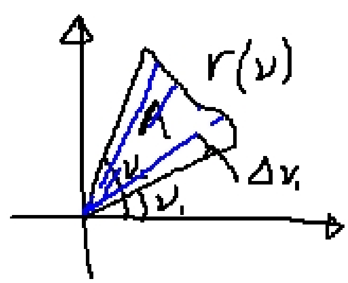
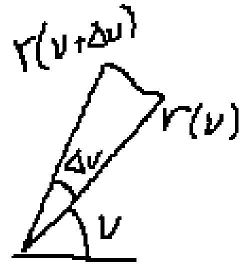
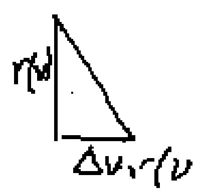
Polära koordinater





$r(\nu+\Delta\nu) \sim r(\nu)$



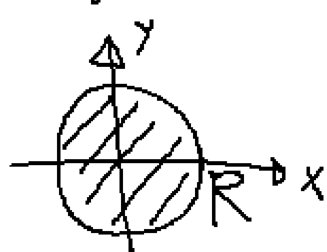
$$A = \lim_{\Delta\nu\to 0} \sum \frac{r^2(\nu_i)}{2}\Delta\nu_i$$

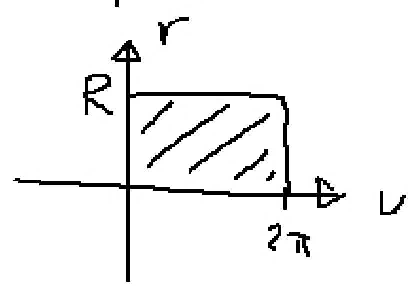
liten arean

$$r(\nu)\,\frac{1}{2}\,r(\nu)\Delta\nu$$

$$= \frac{1}{2}\int_{\nu_1}^{\nu_2} r^2(\nu)\,d\nu$$

Ex: cirkelskivans area. Vi tar en cirkel med radie r och centrerad i origo och beräknar arean innanför.





$$A = \frac{1}{2}\int_0^{2\pi} r^2(v)\,dv = \frac{1}{2}\int_0^{2\pi} R^2\,dv = \frac{R^2}{2}\int_0^{2\pi} dv =$$

$$= \frac{2\pi}{2}R^2 = \pi R^2$$

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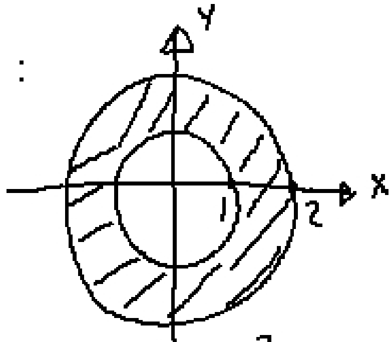
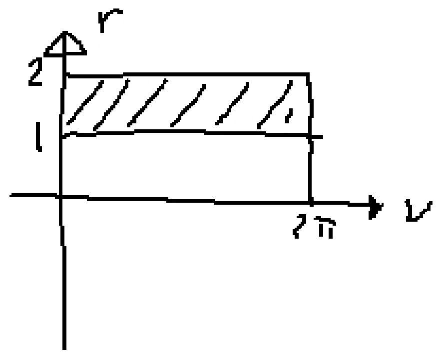
Ex:

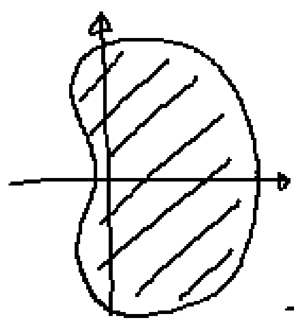




$$A = \frac{1}{2}\int_0^{2\pi} r_1(v)^2 - r_2(v)^2 \, dv =$$

$$= \frac{1}{2}\int_0^{2\pi} 2^2 - 1^2 \, dv = \frac{3}{2}\int_0^{2\pi} dv = 3\pi$$

Ex:



$$r(v) = 3 + 2\cos v$$

$$A = \frac{1}{2}\int_0^{2\pi} (3 + 2\cos v)^2 \, dv =$$

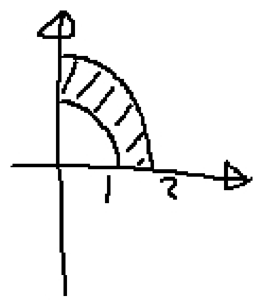
$$= \frac{1}{2}\int_0^{2\pi} 9 + 12\cos v + 4\cos^2 v \, dv =$$

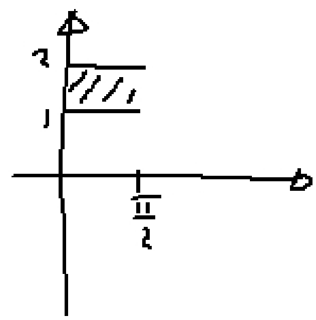
$$= \frac{1}{2}\int_0^{2\pi} 9 + 12\cos v + 4\left(\frac{1+\cos 2v}{2}\right) dv =$$

$$= \frac{1}{2}\Big[9v + 12\sin v + 2v + \sin 2v\Big]_0^{2\pi} =$$

$$= 11\pi$$

Ex



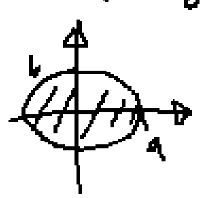


$$A = \frac{1}{2}\int_0^{\frac{\pi}{2}} 2^2 - 1^2 \, dv = \frac{3\pi}{4}$$

Ex: (Arean av en ellips)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad y = \pm b\sqrt{1 - \frac{x^2}{a^2}}$$

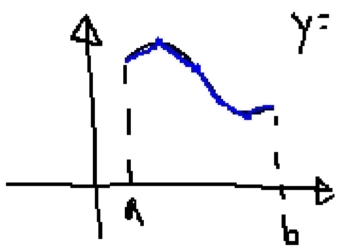
$$\text{Arean} = 2b \int_{-a}^{a} \sqrt{1 - \frac{x^2}{a^2}} \, dx = \left[\begin{array}{l} x = a\cos v \\ dx = -a\sin v \, dv \end{array}\right] =$$



$$= -2ba \int_{\pi}^{0} \sin^2 v \, dv = -2ba \int_{\pi}^{0} \frac{1 - \cos 2v}{2} \, dv =$$

$$= ba\left[v - \frac{\sin 2v}{2}\right]_0^{\pi} = \pi ab$$

Båglängd

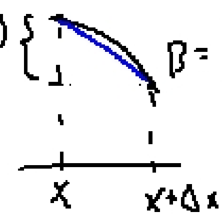


$y = f(x)$

$A = (x, f(x))$

$f(x+\Delta x) - f(x)$

$B = (x+\Delta x, f(x+\Delta x))$



$$|AB| = \sqrt{(x+\Delta x - x)^2 + (f(x+\Delta x) - f(x))^2}$$

$$= \sqrt{(\Delta x)^2 + (f(x+\Delta x) - f(x))^2} =$$

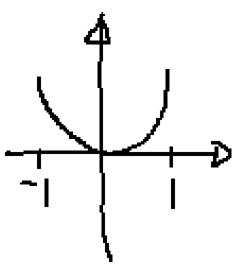
$$= \sqrt{1 + \left(\frac{f(x+\Delta x) - f(x)}{\Delta x}\right)^2}\,\Delta x$$

När $\Delta x \to 0$ får vi:

$\sqrt{1+(f'(x))^2}\,dx$ så längden

blir $$S = \int_a^b \sqrt{1+(f'(x))^2}\,dx$$

Ex: Beräkna längden av kurvan $y = x^2$ då $-1 \leq x \leq 1$.



$$\int^{1} \sqrt{1+(2x)^2}\,dx = \left[\begin{array}{l} 2x = \sinh t \\ dx = \frac{1}{2}\cosh t\,dt \end{array}\right] =$$

$$= \int_{\operatorname{arcsinh}(-2)}^{\operatorname{arcsinh}(2)} \frac{\cosh^2 t}{2}\,dt =$$

$$= \frac{1}{2}\int_{\operatorname{arcsh}(-2)}^{\operatorname{arcsh} 2}\left(\frac{e^{t}+e^{-t}}{2}\right)^{2}dt = \frac{1}{2}\int_{\operatorname{arcsh}(-2)}^{\operatorname{arcsh} 2}\frac{e^{2t}}{4}+\frac{1}{2}+\frac{e^{-2t}}{4}\,dt =$$

$$= \frac{1}{2}\left[\frac{e^{2t}}{8}+\frac{t}{2}-\frac{e^{-2t}}{8}\right]_{\operatorname{arcsh}(-2)}^{\operatorname{arcsh} 2} =$$

$$= \frac{1}{2}\left[\frac{t}{2}+\frac{\sinh 2t}{4}\right]_{\operatorname{arcsh}(-2)}^{\operatorname{arcsh} 2} =$$

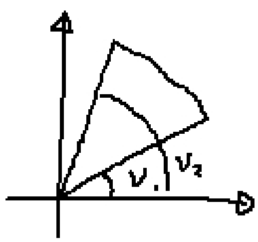
$$= \frac{1}{2}\left(\frac{\operatorname{arcsh} 2}{2}-\frac{\operatorname{arcsh}(-2)}{2}\right)+\frac{1}{4}\left[\sinh t\cosh t\right]_{\operatorname{arcsh}(-2)}^{\operatorname{arcsh} 2} =$$

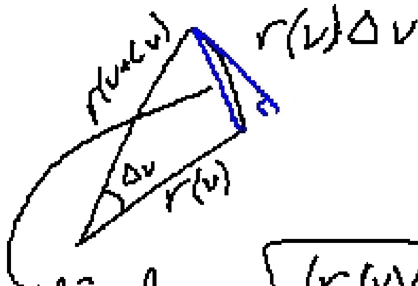
$$= \frac{\operatorname{arcsh} 2}{2}+\frac{1}{4}\Big(2\cosh(\operatorname{arcsh} 2)-(-2)\cosh(\operatorname{arcsh}(-2))\Big) =$$

$$= \frac{\operatorname{arcsh} 2}{2}+\cosh(\operatorname{arcsh} 2) =$$

$$= \frac{\operatorname{arcsh} 2}{2}+\sqrt{1+\sinh^{2}(\operatorname{arcsh} 2)} = \frac{\operatorname{arcsh} 2}{2}+\sqrt{1+4}$$

polärt





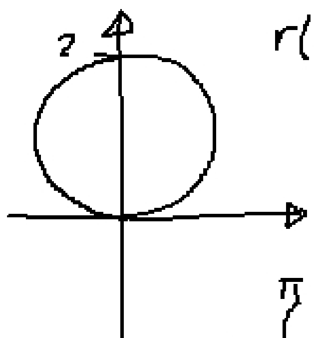
längden $\sim \sqrt{(r(v)\Delta v)^2 + (r(v+\Delta v)-r(v))^2}$

$$= \sqrt{r(v)^2 + \left(\frac{r(v+\Delta v)-r(v)}{\Delta v}\right)^2}\;\Delta v$$

$$S = \int_{v_1}^{v_2} \sqrt{r(v)^2 + (r'(v))^2}\,dv$$

Ex: Cirkelns omkrets, radie R

$$\int_0^{2\pi} \sqrt{R^2 + 0^2}\,dv = R\int_0^{2\pi} 1\,dv = 2\pi R$$

Ex: 

$r(v) = 2\sin v$

$$\int_0^{\pi} \sqrt{r(v)^2 + (r'(v))^2}\, dv =$$

$$= \int_0^{\pi} \sqrt{4\sin^2 v + 4\cos^2 v}\, dv =$$

$$= \int_0^{\pi} \sqrt{4}\, dv = 2\int_0^{\pi} dv = 2\pi$$

816 a) Skissera grafen till

$$y = \frac{x^3+1}{x^2+7x} = x-7+\frac{49x+1}{x^2+7x}$$

$$\begin{array}{rl}
 & x-7 \\
x^2+7x \,\big) & x^3+0x^2+0x+1 \\
- & \underline{x^3+7x^2} \\
 & -7x^2+0x \\
- & \underline{-7x^2-49x} \\
 & 49x+1
\end{array}$$

Lodrät asymptot

$x^2+7x=0$

$x=0$ eller $y=-7$

$\frac{49x+1}{x^2+7x} \to 0$ när $x \to \pm\infty$

Så $x-7$ är en sned asymptot.