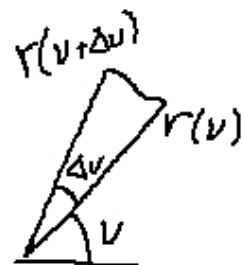
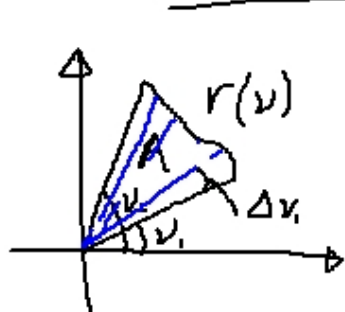
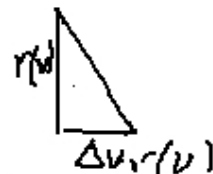


Polära koordinater



$$r(v+\Delta v) \sim r(v)$$



$$A = \lim_{\Delta v \rightarrow 0} \sum \frac{r^2(v_i)}{2} \Delta v_i$$

$$= \frac{1}{2} \int_{\varphi_1}^{\varphi_2} r^2(v) dv$$

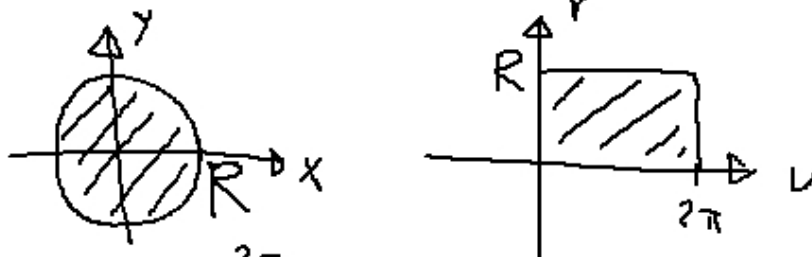
litet arean

$$r(v) \frac{1}{2} r(v) \Delta v$$

Ex: cirkelstivans area. Vi tar en cirkel med radie r och

Föreläsning 29, sid 2

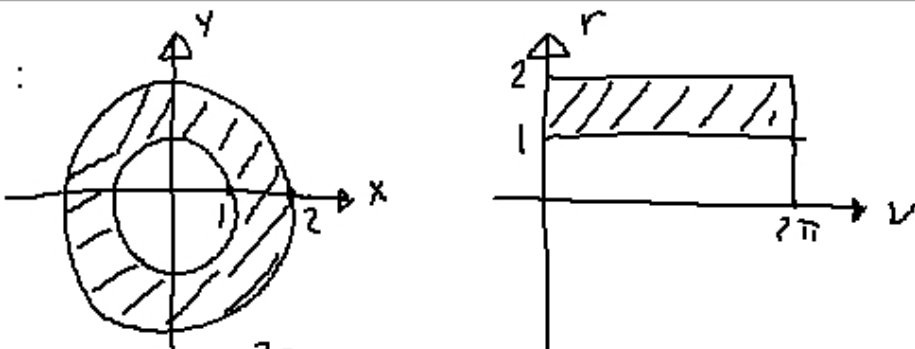
centered i origo och beräkna
arean innan för.



$$A = \frac{1}{2} \int_0^{2\pi} r^2(v) dv = \frac{1}{2} \int_0^{2\pi} R^2 dv = \frac{R^2}{2} \int_0^{2\pi} dv =$$
$$\frac{2\pi R^2}{2} = \pi R^2$$

Föreläsning 29, sid 3

Ex:

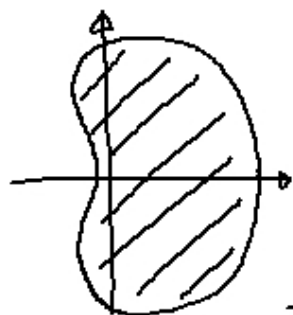


The diagram on the left shows a shaded annulus in the xy -plane. The inner circle has a radius of 1, and the outer circle has a radius of 2. The region between the two circles is shaded with diagonal lines. The x and y axes are shown, with the origin at the center. The inner radius is labeled '1' and the outer radius is labeled '2'.

The diagram on the right shows the corresponding region in the r - v plane. The vertical axis is labeled r and has tick marks at 1 and 2. The horizontal axis is labeled v and has a tick mark at 2π . The region is a rectangle with r ranging from 1 to 2 and v ranging from 0 to 2π . The area between $r=1$ and $r=2$ is shaded with diagonal lines.

$$A = \frac{1}{2} \int_0^{2\pi} r_1(v)^2 - r_2(v)^2 dv =$$
$$= \frac{1}{2} \int_0^{2\pi} 2^2 - 1^2 dv = \frac{3}{2} \int_0^{2\pi} dv = 3\pi$$

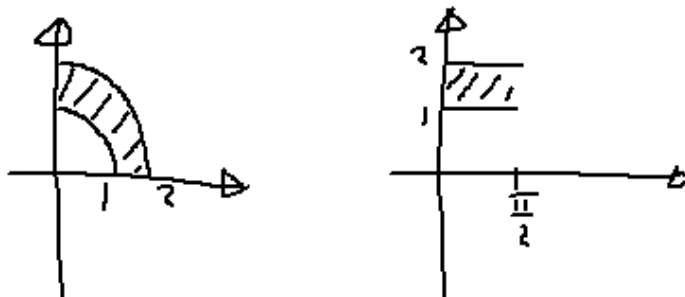
Ex:



$$r(\nu) = 3 + 2 \cos \nu$$


$$\begin{aligned} A &= \frac{1}{2} \int_0^{2\pi} (3 + 2 \cos \nu)^2 d\nu = \\ &= \frac{1}{2} \int_0^{2\pi} 9 + 12 \cos \nu + 4 \cos^2 \nu d\nu = \\ &= \frac{1}{2} \int_0^{2\pi} 9 + 12 \cos \nu + 4 \frac{(1 + \cos 2\nu)}{2} d\nu = \\ &= \frac{1}{2} \left[9\nu + 12 \sin \nu + 2\nu + \sin 2\nu \right]_0^{2\pi} = \\ &= 11\pi \end{aligned}$$

Ex



$$A = \frac{1}{2} \int_0^{\frac{\pi}{2}} 2^2 - 1^2 dv = \frac{3\pi}{4}$$

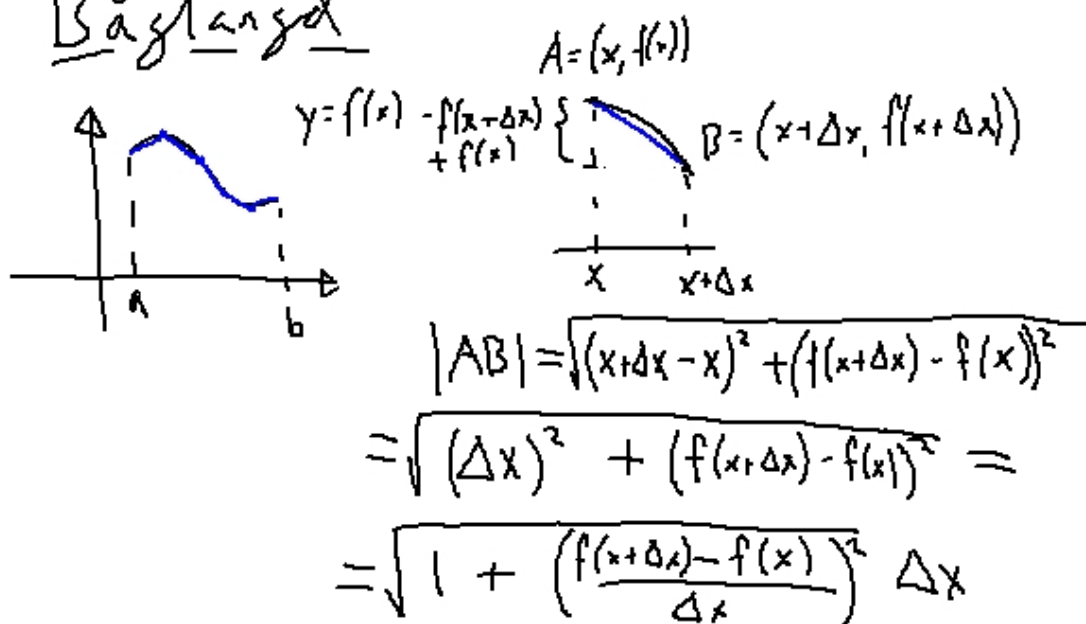
Ex: (Area av en ellips)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad y = \pm b \sqrt{1 - \frac{x^2}{a^2}}$$


$$\begin{aligned} \text{Area} &= 2b \int_{-a}^a \sqrt{1 - \frac{x^2}{a^2}} dx = \left[\begin{array}{l} x = a \cos v \\ dx = -a \sin v dv \end{array} \right] = \\ &= -2ba \int_{\pi}^0 \sin^2 v dv = -2ba \int_{\pi}^0 \frac{1 - \cos 2v}{2} dv = \end{aligned}$$

$$= ba \left[1 - \frac{\sin 2v}{2} \right]_0^\pi = \pi ab$$

Båglängd



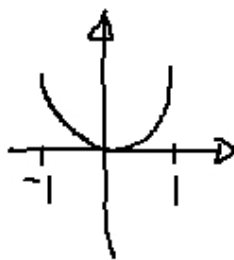
När $\Delta x \rightarrow 0$ får vi:

$$\sqrt{1 + (f'(x))^2} dx \text{ så längden}$$

blir

$$S = \int_a^b \sqrt{1 + (f'(x))^2} dx$$

Ex: Beräkna längden av kurvan
 $y = x^2$ då $-1 \leq x \leq 1$.



$$\int_{\text{arcsinh}(2)}^1 \sqrt{1 + (2x)^2} dx = \left[\begin{array}{l} 2x = \sinh t \\ dx = \frac{1}{2} \cosh t dt \end{array} \right] =$$

$$= \int_{\text{arcsinh}(2)}^1 \frac{\cosh^2 t}{2} dt =$$

Föreläsning 29, sid 8

$$\begin{aligned}
 &= \frac{1}{2} \int_{\operatorname{arcsinh}(-2)}^{\operatorname{arcsinh} 2} \left(\frac{e^t + e^{-t}}{2} \right)^2 dt = \frac{1}{2} \int_{\operatorname{arcsinh}(-2)}^{\operatorname{arcsinh} 2} \frac{e^{2t}}{4} + \frac{1}{2} + \frac{e^{-2t}}{4} dt \\
 &= \frac{1}{2} \left[\frac{e^{2t}}{8} + \frac{t}{2} - \frac{e^{-2t}}{8} \right]_{\operatorname{arcsinh}(-2)}^{\operatorname{arcsinh} 2} = \\
 &= \frac{1}{2} \left[\frac{t}{2} + \frac{\sinh 2t}{4} \right]_{\operatorname{arcsinh}(-2)}^{\operatorname{arcsinh} 2} = \\
 &= \frac{1}{2} \left(\frac{\operatorname{arcsinh} 2}{2} - \frac{\operatorname{arcsinh}(-2)}{2} \right) + \frac{1}{4} \left[\sinh t \cosh t \right]_{\operatorname{arcsinh}(-2)}^{\operatorname{arcsinh} 2} = \\
 &= \frac{\operatorname{arcsinh} 2}{2} + \frac{1}{4} \left(2 \cosh(\operatorname{arcsinh} 2) - (-2) \cosh(\operatorname{arcsinh}(-2)) \right) = \\
 &= \frac{\operatorname{arcsinh} 2}{2} + \cosh(\operatorname{arcsinh} 2) = \\
 &= \frac{\operatorname{arcsinh} 2}{2} + \sqrt{1 + \sinh^2(\operatorname{arcsinh} 2)} = \frac{\operatorname{arcsinh} 2}{2} + \sqrt{1 + 4}
 \end{aligned}$$

Föreläsning 29, sid 9



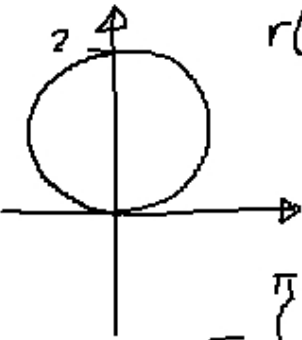
längden $\sim \sqrt{(r(v)\Delta v)^2 + (r(v+\Delta v) - r(v))^2}$
 $= \sqrt{r(v)^2 + \left(\frac{r(v+\Delta v) - r(v)}{\Delta v}\right)^2} \Delta v$

$$S = \int_{v_1}^{v_2} \sqrt{r(v)^2 + (r'(v))^2} dv$$

Ex: Cirkelns omkrets, radie R

$$\int_0^{2\pi} \sqrt{R^2 + 0^2} dv = R \int_0^{2\pi} 1 dv = 2\pi R$$

Föreläsning 29, sid 10

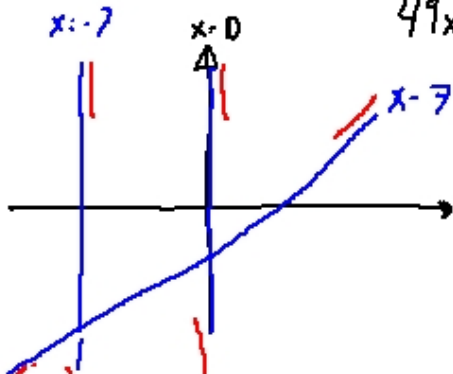
Γ_x :  $r(v) = 2 \sin v$

$$\int_0^{\pi} \sqrt{r(v)^2 + (r'(v))^2} dv =$$
$$= \int_0^{\pi} \sqrt{4 \sin^2 v + 4 \cos^2 v} dv =$$
$$= \int_0^{\pi} \sqrt{4} dv = 2 \int_0^{\pi} dv = 2\pi$$

816 a) Skissera grafen till

$$y = \frac{x^3 + 1}{x^2 + 7x} = x - 7 + \frac{49x + 1}{x^2 + 7x}$$

$$\begin{array}{r} x-7 \\ x^2+7x \overline{) x^3+0x^2+0x+1} \\ - \quad x^3+7x^2 \\ \hline -7x^2+0x \\ - \quad -7x^2-49x \\ \hline 49x+1 \end{array}$$



Lodrat asymptot

$$x^2 + 7x = 0$$

$$x = 0 \text{ eller } x = -7$$

$$\frac{49x+1}{x^2+7x} \rightarrow 0 \text{ när } x \rightarrow \pm\infty$$

Så $x-7$ är en sned asymptot.