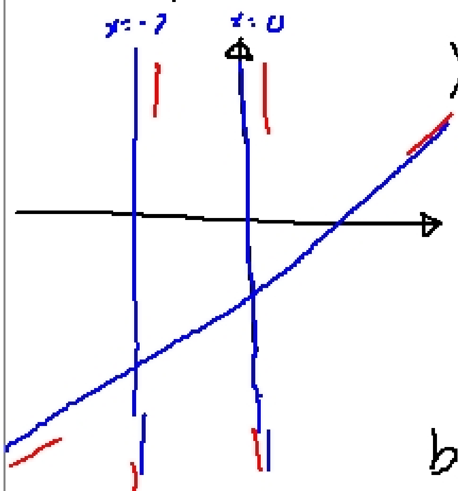
Forts 816 a)



$$y = x - 7 + \frac{49x+1}{x^2+7x}$$

$$y' = 1 + D\left(\frac{49x+1}{x^2+7x}\right) =$$

$$= \frac{x^4+14x^3-2x-7}{(x^2+7x)^2}$$

blir krångligt. Vi partialbråksuppdelar

$$\frac{49x+1}{x(x+7)} = \frac{\frac{1}{7}}{x} + \frac{\frac{-343+1}{-7}}{x+7} = \frac{\frac{1}{7}}{x} + \frac{\frac{342}{7}}{x+7}$$

$$y = x - 7 + \frac{1}{7}\left(\frac{1}{x} + \frac{342}{x+7}\right)$$

$$y' = 1 + \frac{1}{7}\left(-\frac{1}{x^2} - \frac{342}{(x+7)^2}\right)$$

$$y'' = \frac{1}{7}\left(\frac{2}{x^3} + \frac{2\cdot 342}{(x+7)^3}\right) = \frac{2}{7}\left(\frac{1}{x^3} + \frac{342}{(x+7)^3}\right)$$

$x > 0 \Rightarrow y'' > 0$

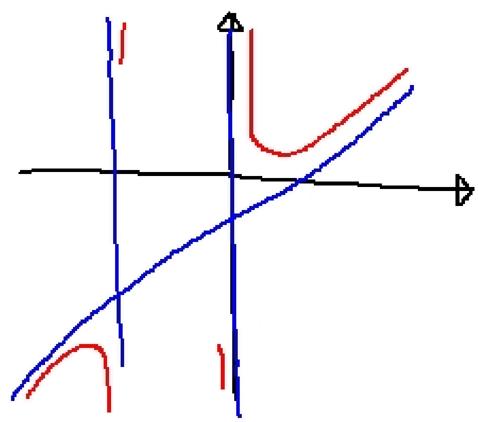
$x < -7 \Rightarrow y'' < 0$

nära $x = -7$, $x > -7$ blir

$y'' > 0$

nära $x = 0$ $x < 0$ blir

$y'' < 0$



$\Rightarrow$ $-y''$ i någon punkt i $(-7, 0)$

$$y'' = \frac{2}{7}\left(\frac{1}{x^3} + \frac{343}{(x+7)^3}\right) \qquad y'(x_1) = y'(x_2) = 0$$

Antag $\dfrac{1}{x_1^3} = -\dfrac{343}{(x_1+7)^3}$ och $\dfrac{1}{x_2^3} = -\dfrac{343}{(x_1+7)^3}$

$$\Rightarrow \quad \frac{(x_1+7)^3}{x_1^3} = \frac{(x_2+7)^3}{x_2^3} = -343$$

$$\Rightarrow \quad \frac{x_1+7}{x_1} = \frac{x_2+7}{x_2} \qquad (\text{för } a^3 = b^3 \Rightarrow a = b)$$

$$\Rightarrow \quad (\cancel{x_1}+7)x_2 = x_1(\cancel{x_2}+7)$$

$$\Rightarrow \quad x_1 = x_2$$

$\Rightarrow$ y'' har bara ett nollställe

	-7	-1		0	
y''	$+$	$+$	0	$-$	$-$
y'	$\nearrow$	$\nearrow$		$\searrow$	$\searrow$

max för y'

Sätter vi in $x=-1$ i y'' får vi $\frac{2}{7}\left(-1+\frac{342}{(-1+7)}\right) > 0$

$\Rightarrow x^3 > -1$

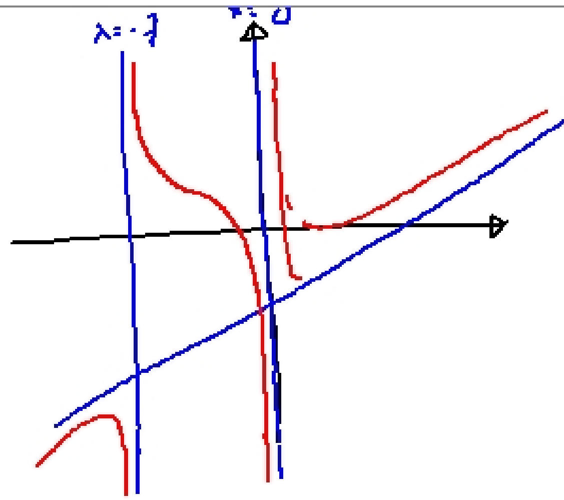
$\Rightarrow y' < 0 \quad -7 < x < 0$

Vi vill att $y' < 0$ för $-7 < x < 0$ så det vi ska visa är att detta maximumet är < 0. I maximipunkten

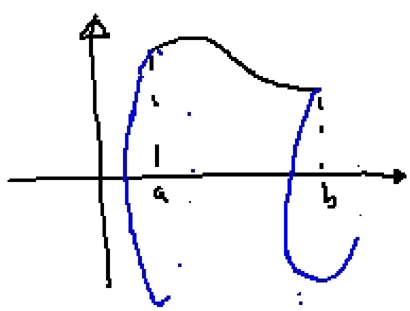
$$y' = 1 - \frac{1}{7}\left(\frac{1}{x^2} + \frac{342}{(x+7)^2}\right) = 1 - \frac{1}{7}\left(-\frac{342x}{(x+7)^3} + \frac{342}{(x+7)^2}\right) =$$

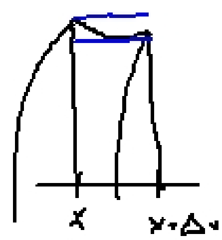
$$= 1 - \frac{342}{7(x+7)^2}\left(1 - \frac{x}{x+7}\right) = 1 - \frac{342}{7(x+7)^2}\left(\frac{x+7-x}{x+7}\right) =$$

$$= 1 - \frac{342}{(x+7)^3} = 1 + \frac{1}{x^3} < 0 \text{ om } x^3 > -1$$


x = -1

Rotationsvolymer





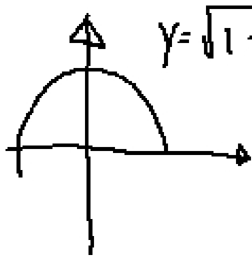
volymen

$A\Delta x$

$= \pi f(x)^2 \Delta x$

$$V = \pi \int_a^b f(x)^2 \, dx$$

Ex (enhetsklotets volym):

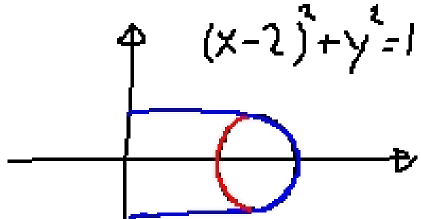


$y = \sqrt{1 - x^2}$

$$V = \pi \int_{-1}^{1} \left(\sqrt{1 - x^2}\right)^2 dx = \pi \int_{-1}^{1} 1 - x^2 \, dx =$$

$$= \pi \left[x - \frac{x^3}{3} \right]_{-1}^{1} = \pi \left(2 - \frac{2}{3} \right) = \frac{4\pi}{3}.$$

Ex: $(x-2)^2+y^2=1 \qquad y=\pm\sqrt{1-(x-2)^2}$



$x=\sqrt{1-y^2}+2$

Rotation kring x-axeln

$$V_x = \pi\int_1^3 \left(\sqrt{1-(x-2)^2}\right)^2 dx = \begin{bmatrix} t=x-2 \\ dt=dx \end{bmatrix} = \pi\int_{-1}^{1} 1-t^2\,dt = \frac{4\pi}{3}$$

Rotation kring y-axeln

$$x=\sqrt{1-y^2}+2 \qquad V_y = \pi\int_{-1}^{1}\left(\sqrt{1-y^2}+2\right)^2 dy =$$

$$=\pi\int_{-1}^{1} 1-y^2+4\sqrt{1-y^2}+4\,dy = \frac{4\pi}{3}+4\pi\int_{-1}^{1}\sqrt{1-y^2}\,dy+8\pi$$

$$=\begin{bmatrix} y=\sin t \\ dy=\cos t\,dt \end{bmatrix} = \frac{4\pi}{3}+8\pi+4\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos^2 t\,dt =$$

$$= \frac{28\pi}{3} + 4\pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1+\cos 2t}{2}\, dt =$$

$$= \frac{28\pi}{3} + 4\pi \left[\frac{t}{2} + \frac{\sin 2t}{4} \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} =$$

$$= \frac{28\pi}{3} + 4\pi \left(\frac{\pi}{4} - \left(-\frac{\pi}{4}\right) + 0 \right) = \frac{28\pi}{3} + 2\pi^2$$

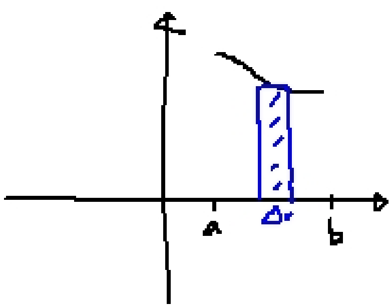
$$V_2 = \pi \int_{-1}^{1} \left(2 - \sqrt{1-y^2}\right)^2 dy = \pi \int_{-1}^{1} 4 - 4\sqrt{1-y^2} + 1 - y^2\, dy =$$

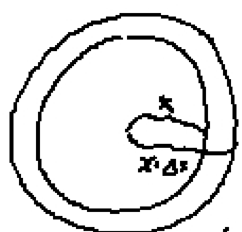
$$= 8\pi - 4\pi \int_{-1}^{1} \sqrt{1-y^2}\, dy + \pi \int_{-1}^{1} 1 - y^2\, dy =$$

$$= 8\pi - 2\pi^2 + \frac{4\pi}{3} = \frac{28\pi}{3} - 2\pi^2$$

$$V = V_1 - V_2 = \frac{28\pi}{3} + 2\pi^2 - \left(\frac{28\pi}{3} - 2\pi^2 \right) = 4\pi^2.$$

Rotation kring y-axeln kan också behandlas genom att ta cylindriska skal





$$\pi(x+\Delta x)^2 y - \pi x^2 y$$
$$= 2\pi x y \Delta x + \pi y (\Delta x)^2$$
$$\sim 2\pi x y \Delta x$$

$$\text{Vol} \quad 2\pi \int_a^b x f(x)\,dx$$

Torusens volym på nytt:

$$(x-2)^2 + y^2 = 1 \qquad y = \pm\sqrt{1-(x-2)^2}$$

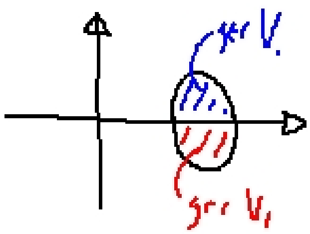
$$y = \sqrt{1-(x-2)^2}, \quad V_1 = 2\pi \int_1^3 x\sqrt{1-(x-2)^2}\,dx = \begin{bmatrix} t = x-2 \\ dt = dx \end{bmatrix} =$$

$$= 2\pi \int_{-1}^{1} (t+2)\sqrt{1-t^2}\,dt = 2\pi \int_{-1}^{1} t\sqrt{1-t^2}\,dt +$$

$$= \pi \int_{-1}^{1} 2t\sqrt{1-t^2}\,dt + 4\pi \int_{-1}^{1} \sqrt{1-t^2}\,dt = \left[-\frac{\pi 2}{3}(1-t^2)^{\frac{3}{2}}\right]_{-1}^{1} + 2\pi^2 =$$

$$= 2\pi^2$$

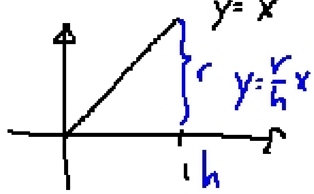
$$y = -\sqrt{1-(x-2)^2} \qquad V_2 = -2\pi \int_1^3 x\sqrt{1-(x-2)^2}\,dx = -V_1$$



$$V = V_1 - V_2 = 2V_1 = 4\pi^2$$

Ex: Volymen hos en kon

$y = x$ roteras kring x-axeln



$$\pi \int_0^1 y^2\,dx = \pi\left[\frac{x^3}{3}\right]_0^1 = \frac{\pi}{3}$$

$$\pi \int_0^h y^2\,dx = \pi \frac{r^2}{h^2}\left[\frac{x^3}{3}\right]_0^h = \frac{\pi r^2 h}{3}$$