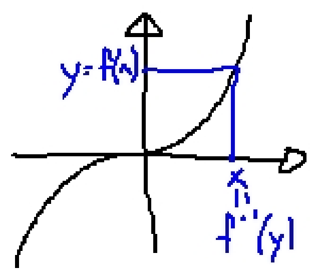
Om f är inverterbar

$$f(x) = y \iff f^{-1}(y) = x$$

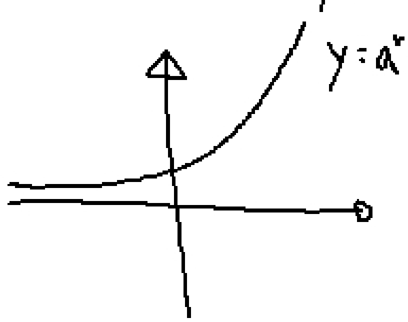


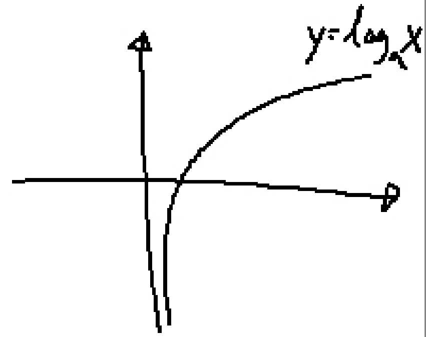
$$(f^{-1})^{-1} = f$$

Ex: Om $a>1$, då är a^x
strängt växande så
den har en inversfunktion.

$$f(x) = a^x = y \Rightarrow f^{-1}(y) = x$$

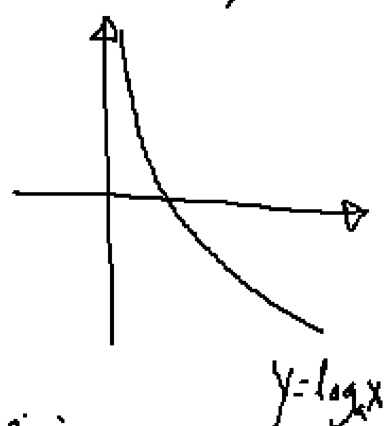
dvs $f^{-1}(y) = \log_a y$.



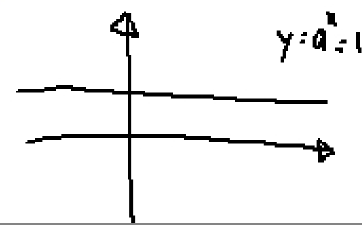


Om $a<1$ då är $f(x)=a^x$ strängt avtagande så f har invers. $f^{-1}(x)=\log_a x$

$\left(a=\frac{1}{b},\ b>1,\ a^x=b^{-x} \right)$



$a=1$



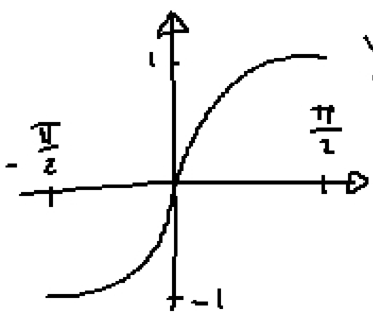
ej inverterbar

Cyklometriska funktioner

(inverser till trigonometriska funktioner)

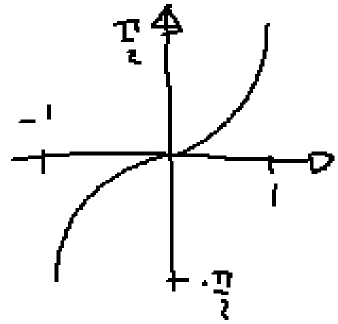
Trigonometriska funktioner är periodiska så de saknar inverser.

För att de ska ha inverser måste begränsa definitionsområdena.



$y = \sin x$

$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

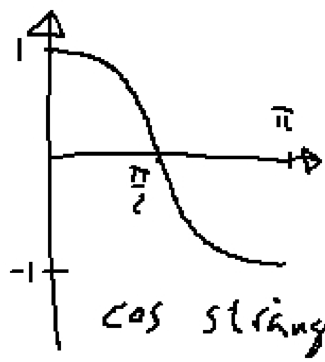


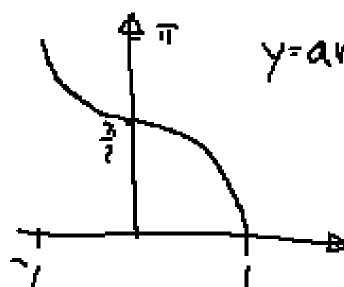
sinus funktionen är strängt växande på intervallet

$y = \arcsin x$ invers till sinus

$y = \cos x$

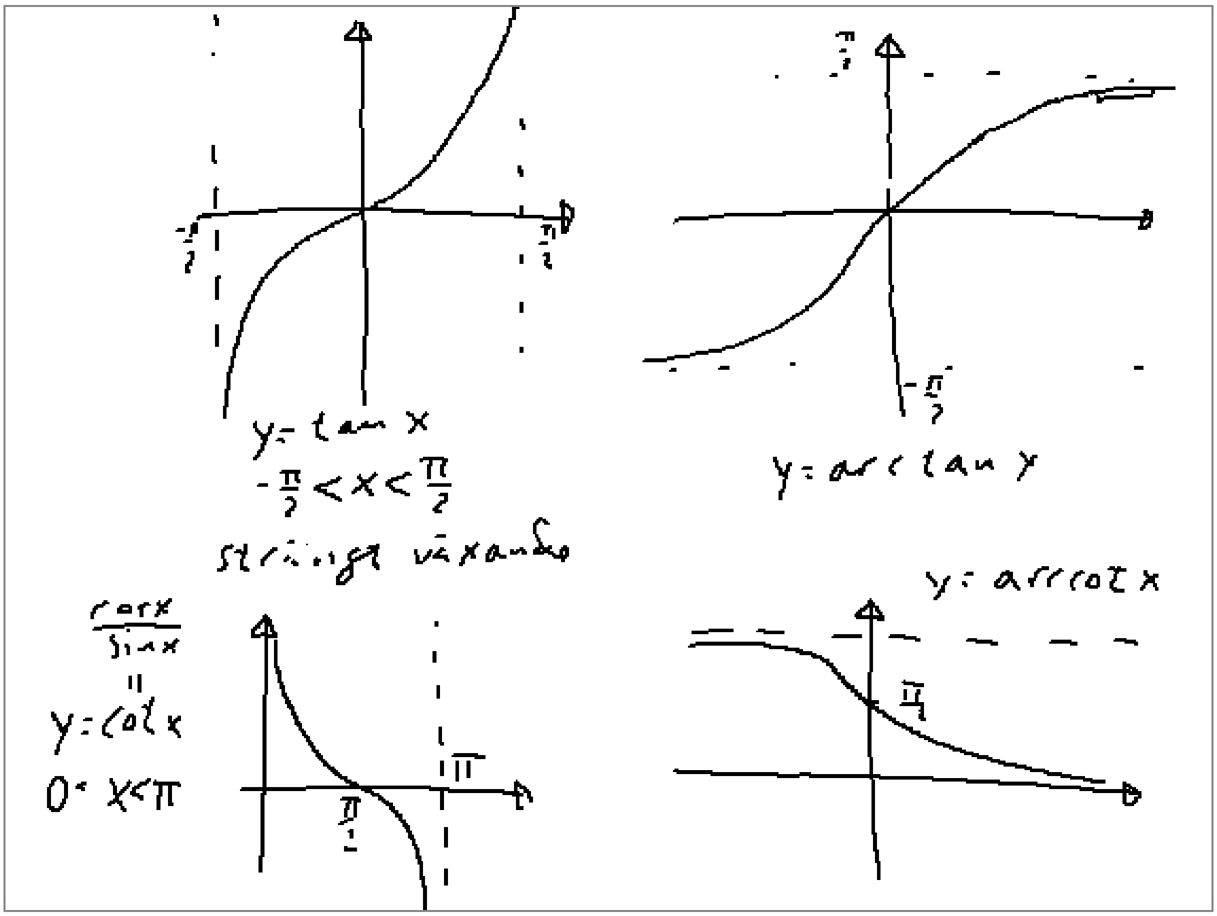
$0 \leq x \leq \pi$





$y = \arccos x$

cos strängt avtagande på intervallet.


y = tan x
-π/2 < x < π/2
strängt växande
y = arctan x
cos x / sin x = y = cot x
0 < x < π
y = arccot x
π/2

$$\sin(\arcsin x) = \cos(\arccos x) = x \quad -1 \le x \le 1$$

$$\tan(\arctan x) = \cot(\operatorname{arccot} x) = x \quad \text{for all } x$$

$$\arcsin(\sin x) = x \qquad -\frac{\pi}{2} \le x \le \frac{\pi}{2}$$

$$\arcsin(\sin \pi) = \arcsin 0 =$$

$$= \arcsin(\sin 0) = 0$$

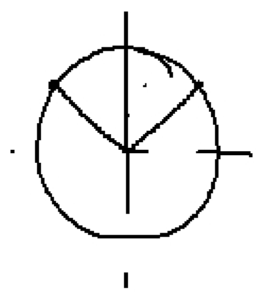
$$\arccos(\cos x) = x \qquad 0 \le x \le \pi$$

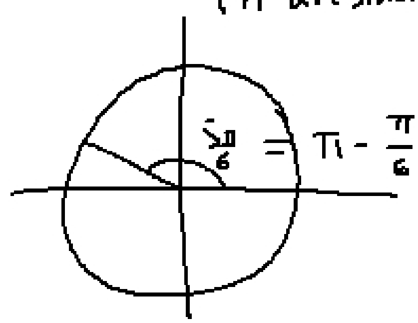
$$\arctan(\tan x) = x \qquad -\frac{\pi}{2} < x < \frac{\pi}{2}$$

$$\operatorname{arccot}(\cot x) = x \qquad 0 < x < \pi$$

Vi har att

$$\sin x = a \iff x = \begin{cases} \arcsin a + 2\pi n \\ \pi - \arcsin a + 2\pi n \end{cases}$$



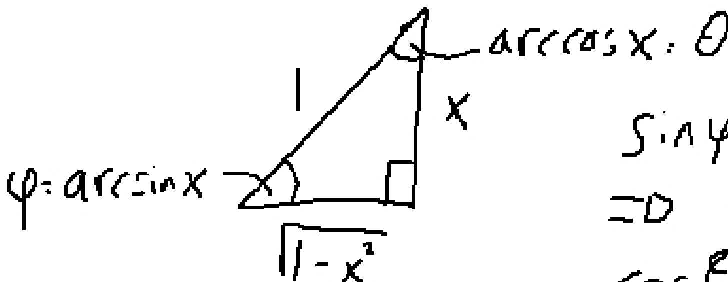


$$\cos x = a \iff x = \pm \arccos a + 2\pi n$$

$$\tan x = a \iff x = \arctan a + \pi n$$

$$\cot x = a \iff x = \operatorname{arccot} a + \pi n$$

Ex: $\arcsin\left(\sin \frac{5\pi}{6}\right) = \arcsin \frac{1}{2} = \frac{\pi}{6}$

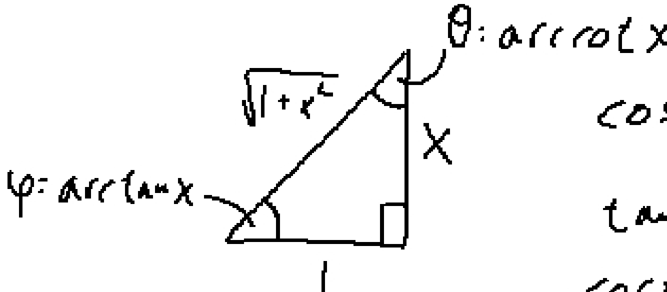


$\sin\varphi = x$

$\Rightarrow \varphi = \arcsin x$

$\cos\theta = x$

$\Rightarrow \theta = \arccos x$



$\cos\varphi = \frac{1}{\sqrt{1+x^2}}$, $\sin\varphi = \frac{x}{\sqrt{1+x^2}}$

$\tan\varphi = x \Rightarrow \arctan x = \varphi$

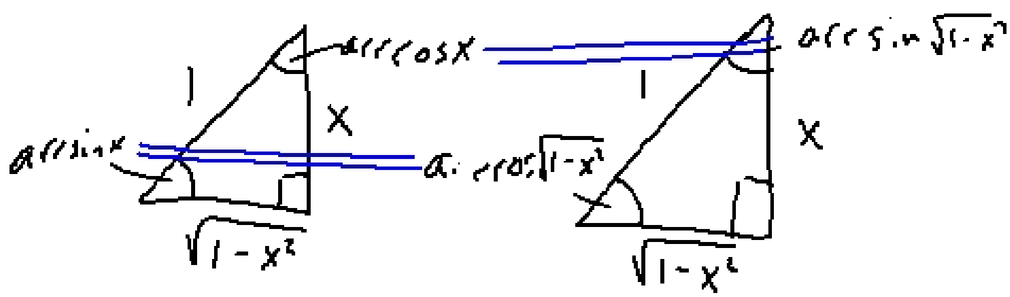
$\cos\theta = \frac{x}{\sqrt{1+x^2}}$, $\sin\theta = \frac{1}{\sqrt{1+x^2}}$

$\cot\theta = x \qquad \theta = \operatorname{arccot} x$

$$\arcsin x + \arccos x = \frac{\pi}{2}$$

$$\arctan x + \operatorname{arccot} x = \frac{\pi}{2}$$

$$\sin(\arccos x) = \sqrt{1-x^2}$$



Ex: Förenkla $\arcsin\frac{4}{5} - \arctan\frac{1}{7}$

Vi börjar med att ta sinus av uttrycket.

$$\sin\left(\arcsin\frac{4}{5} - \arctan\frac{1}{7}\right) =$$

$$= \sin\left(\arcsin\frac{4}{5}\right)\cos\left(\arctan\frac{1}{7}\right) -$$

$$\sin\left(\arctan\frac{1}{7}\right)\cos\left(\arcsin\frac{4}{5}\right) =$$

$$= \frac{4}{5}\cos\left(\arctan\frac{1}{7}\right) - \sin\left(\arctan\frac{1}{7}\right)\sqrt{1-\left(\frac{4}{5}\right)^2}$$

[Övn: Visa att $\sin(\arctan x) = \dfrac{x}{\sqrt{1+x^2}}$

och $\cos(\arctan x) = \dfrac{1}{\sqrt{1+x^2}}$]

$$= \frac{4}{5}\frac{1}{\sqrt{1+\left(\frac{1}{7}\right)^2}} - \frac{\frac{1}{7}}{\sqrt{1+\left(\frac{1}{7}\right)^2}}\sqrt{1-\left(\frac{4}{5}\right)^2} = \frac{4}{5}\frac{7}{\sqrt{50}} - \frac{1}{\sqrt{50}}\frac{3}{5}$$

$$= \frac{28-3}{5\sqrt{50}} = \frac{25}{5\sqrt{50}} = \frac{5}{\sqrt{50}} = \frac{5}{\sqrt{2\cdot 25}} = \frac{5}{\sqrt{2}\cdot\sqrt{25}} = \frac{5}{\sqrt{2}\cdot 5} = \frac{1}{\sqrt{2}}$$

$$\sin\alpha = \frac{1}{\sqrt{2}}$$

$$\alpha = \begin{cases} \frac{\pi}{4} + 2\pi n \\ \frac{3\pi}{4} + 2\pi n \end{cases}$$

$$\alpha = \arcsin\frac{4}{5} - \arctan\frac{1}{7}$$

$$0 \leq \arcsin\frac{4}{5} \leq \frac{\pi}{2} \qquad \frac{4}{5} > 0$$

$$0 \leq \arctan\frac{1}{7} < \frac{\pi}{2} \qquad \frac{1}{7} > 0$$

$$-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$$

$\Rightarrow$ SVAR: $\arcsin\frac{4}{5} - \arctan\frac{1}{7} = \frac{\pi}{4}$.