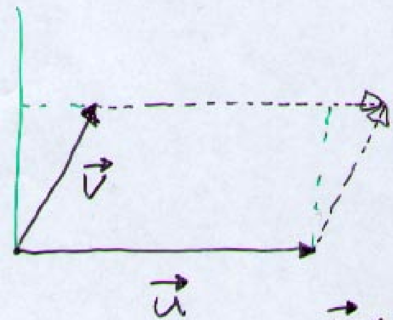
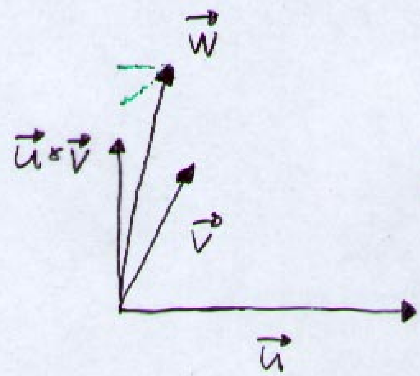
1



$$\vec{u}\cdot(\vec{u}\times\vec{v})=0$$
$$\vec{v}\cdot(\vec{u}\times\vec{v})=0$$

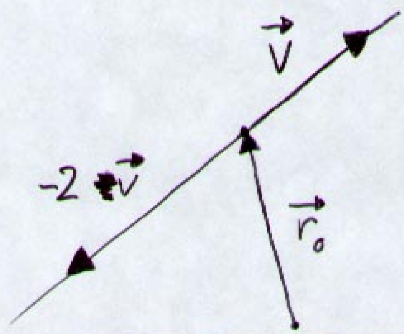
$$\vec{u}\times(\vec{v}+k\vec{u})=\vec{u}\times\vec{v}$$
$$\vec{u}\times\vec{v}=\vec{u}\times\vec{v}^{\perp}_{\vec{u}}$$



$$\vec{w}\cdot(\vec{u}\times\vec{v})=$$
$$\vec{v}\cdot(\vec{w}\times\vec{u})=$$
$$\vec{u}\cdot(\vec{v}\times\vec{w})$$

$$(\vec{w}+k\vec{u})\cdot(\vec{u}\times\vec{v})=\vec{w}\cdot(\vec{u}\times\vec{v})$$
$$(\vec{w}+l\vec{v})\cdot(\vec{u}\times\vec{v})=\vec{w}\cdot(\vec{u}\times\vec{v})$$
$$\vec{w}\cdot(\vec{u}\times\vec{v})=\vec{w}_{\vec{u}\times\vec{v}}\cdot(\vec{u}\times\vec{v})$$

Räta linjer



$$\vec{r} = \vec{r}_0 + t\vec{v}$$

linjens ekvation på vektorform

$$\vec{r} = (x, y)$$

$$\vec{r}_0 = (x_0, y_0)$$

$$\vec{v} = (a, b)$$

$$\begin{cases} x = x_0 + at \\ y = y_0 + bt \end{cases}$$

3

$$\vec{r} = (x, y, z)$$

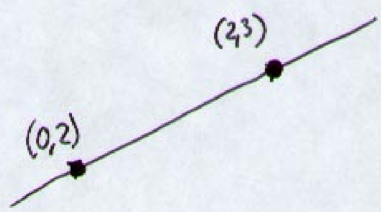
$$\vec{r}_0 = (x_0, y_0, z_0)$$

$$\vec{v} = (a, b, c)$$

$$\boxed{\begin{cases} x = x_0 + at \\ y = y_0 + bt \\ z = z_0 + ct \end{cases}}$$

linjens ekvation på parameterform

Exempel:



$$\vec{r}_0 = (0, 2)$$

$$\vec{v} = (2,3) - (0,2) = (2,1)$$

④

$$\vec{r} = (0,2) + t(2,1)$$

$$\begin{cases} x = 2t \\ y = 2 + t \end{cases}$$

– – – – –

$$\vec{r}_0 = (2,3)$$

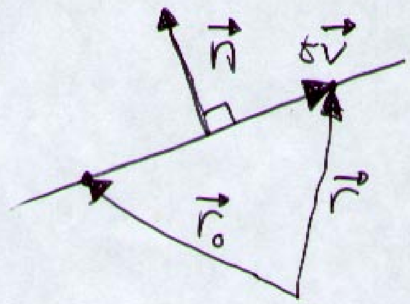
$$\vec{v} = (0,2) - (2,3) = (-2,-1)$$

$$\vec{r} = (2,3) + s(-2,-1)$$

$$\begin{cases} x = 2 - 2s \\ y = 3 - s \end{cases}$$

$$t = \frac{x}{2} = 1 - s$$

$$\begin{cases} 2t = 2 - 2s \\ 2 + t = 3 - s \end{cases}$$



$$(\vec{r}-\vec{r}_0)\cdot\vec{n}=0$$

$$\vec{r}=(x,y)$$
$$\vec{r}_0=(x_0,y_0)$$
$$\vec{n}=(A,B)$$

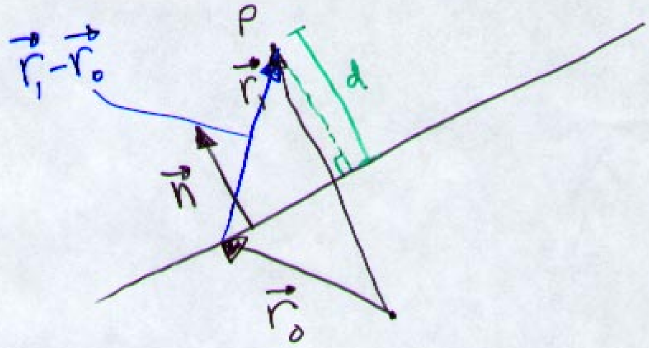
$$0=(x-x_0,y-y_0)\cdot(A,B)=A(x-x_0)+B(y-y_0)$$

Sats 3.1 Om en linje har ekvationen

$$Ax+By+C=0$$

så är vektorn, (A,B), dess normal vektor.

⑥



$$d = \frac{|(\vec{r}_1 - \vec{r}_0) \cdot \vec{n}|}{|\vec{n}|}$$

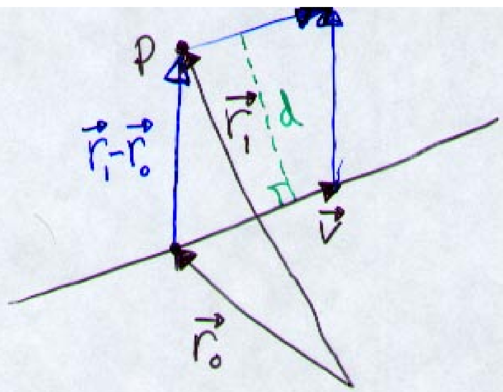
$$\vec{r}_1 = (x_1, y_1)$$

$$\vec{r}_0 = (x_0, y_0)$$

$$\vec{n} = (A, B)$$

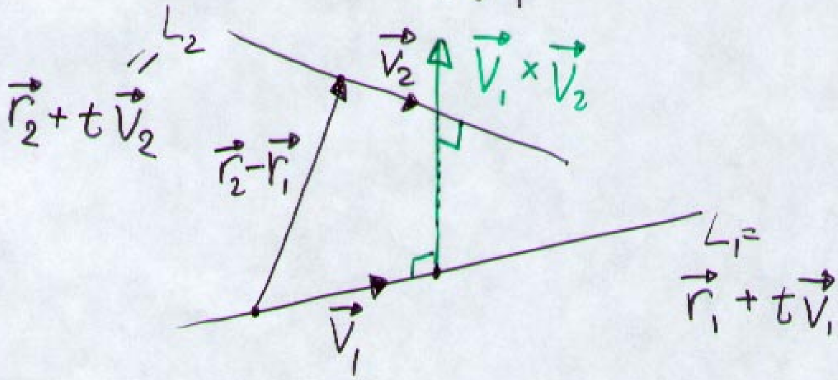
$$d = \frac{|Ax_1 + By_1 + C|}{|(A,B)|}$$

$$C = -Ax_0 - By_0$$

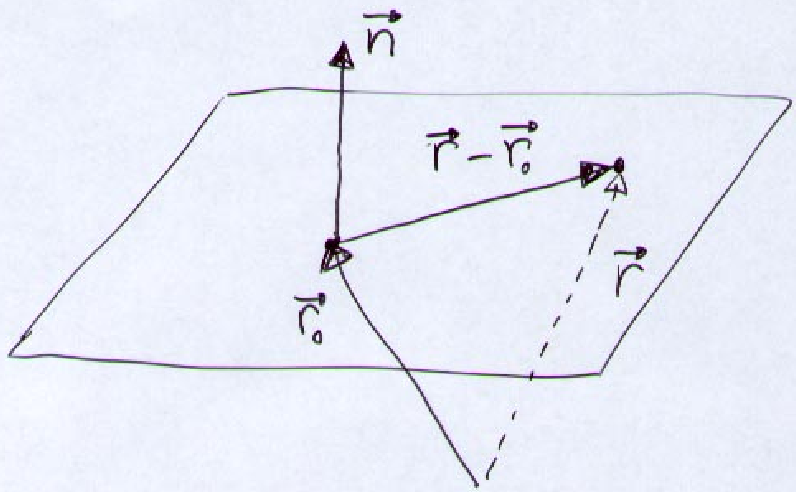


$$d \cdot |\vec{v}| = |(\vec{r}_1 - \vec{r}_0) \times \vec{v}|$$

$$d = \frac{|(\vec{r}_1 - \vec{r}_0) \times \vec{v}|}{|\vec{v}|}$$



$$d = \frac{|(\vec{r}_2 - \vec{r}_1) \cdot (\vec{v}_1 \times \vec{v}_2)|}{|(\vec{v}_1 \times \vec{v}_2)|}$$

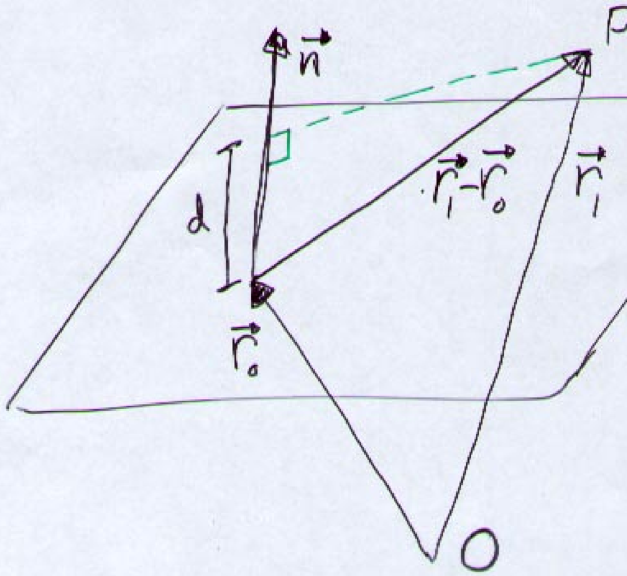


$$(\vec{r}-\vec{r}_0)\cdot\vec{n}=0$$

$$\vec{r}=(x,y,z)$$
$$\vec{r}_0=(x_0,y_0,z_0)$$
$$\vec{n}=(A,B,C)$$

$$A(x-x_0)+B(y-y_0)+C(z-z_0)=0$$

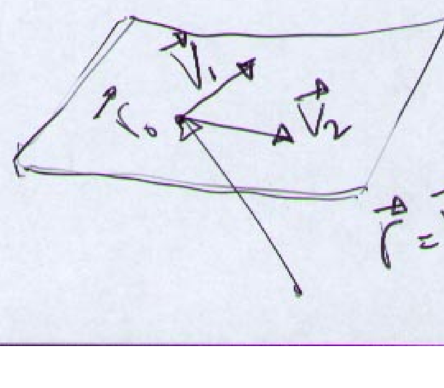
Sats 3.3 Om ett plan har ekvationen $Ax+By+Cz+D=0$ så är (A,B,C) dess normalvektor.



$$d = \frac{|(\vec{r}_1 - \vec{r}_0) \cdot \vec{n}\,|}{|\vec{n}|}$$

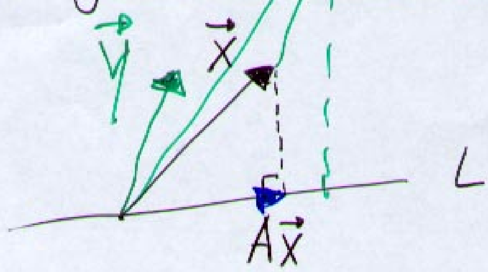
$$\vec{r}_1 = (x_1, y_1, z_1)$$

$$d = \frac{|\,Ax_1 + By_1 + Cz_1 + D\,|}{\sqrt{A^2 + B^2 + C^2}}$$

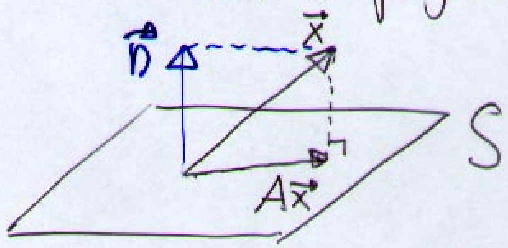


$$\vec{r} = \vec{r}_0 + s\vec{V}_1 + t\vec{V}_2$$

Linjära avbildningar

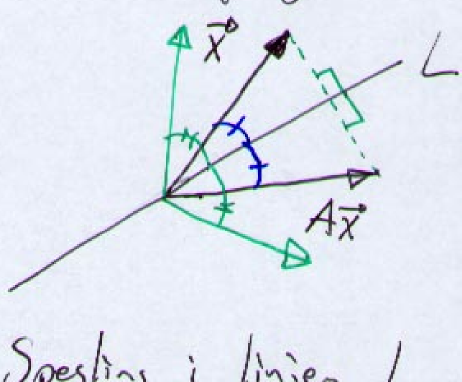


vinkelrät projektion på L



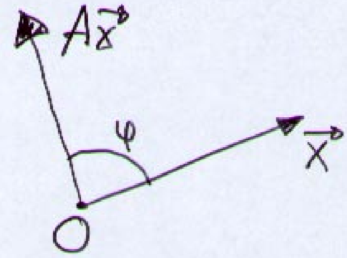
$$A\vec{x} = \vec{x} - \frac{\vec{x}\cdot\vec{n}}{|\vec{n}|^2}\vec{n}$$

vinkelrät projektion på S

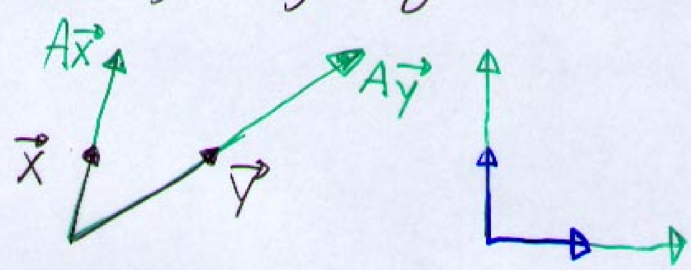


$$|\vec{x}| = |A\vec{x}|$$

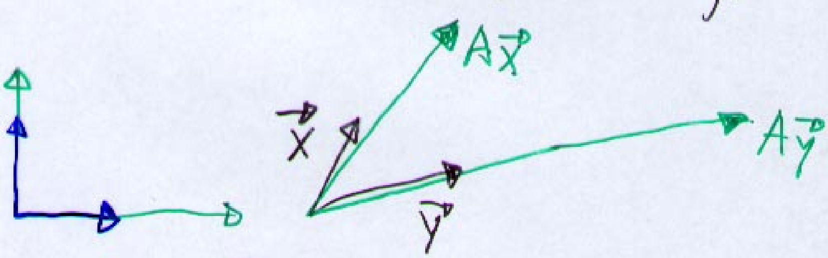
Spegling i linjen L



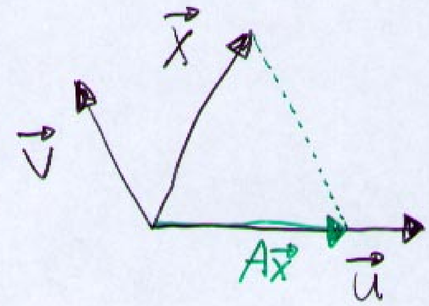
Vridning kring origo



Dilatation, skalning.



Töjning



Projektion på $\vec{u}$ längs $\vec{v}$

Def 3.1: En avbildning A kallas linjär om

$$A(\vec{x}+\vec{y}) = A\vec{x} + A\vec{y}$$
$$A(k\vec{x}) = kA\vec{x}$$

Exempel: $A\vec{x} = |\vec{x}|\,\vec{x}$

är inte linjär.

Låt $\{\vec{e}_1, \vec{e}_2\}$ vara (13)

en bas.

$$\vec{x} = x_1\vec{e}_1 + x_2\vec{e}_2$$

$$A\vec{x} = A(x_1\vec{e}_1) + A(x_2\vec{e}_2) =$$

$$= x_1 A\vec{e}_1 + x_2 A\vec{e}_2$$

Avbildningen bestäms alltså av vad den gör med bas vektorerna.

$$A\vec{e}_1 = a_{11}\vec{e}_1 + a_{21}\vec{e}_2$$

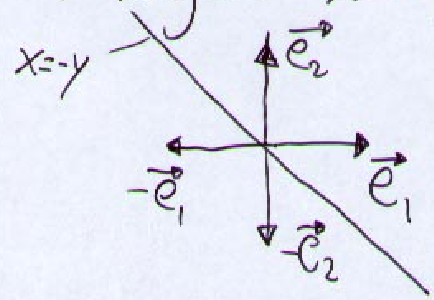
$$A\vec{e}_2 = a_{12}\vec{e}_1 + a_{22}\vec{e}_2$$

Matrisen

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

bestämmer avbildningen A fullständigt. (om man känner basen $\{\vec{e}_1, \vec{e}_2\}$)

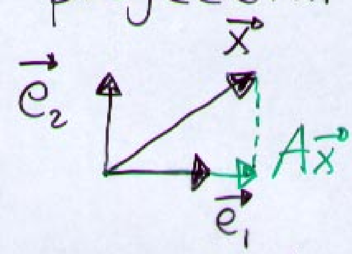
Exempel: Låt A vara spegling i linjen $x=-y$.



$A\vec{e}_1 = -\vec{e}_2$, $A\vec{e}_2 = -\vec{e}_1$

$$\mathbb{A} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

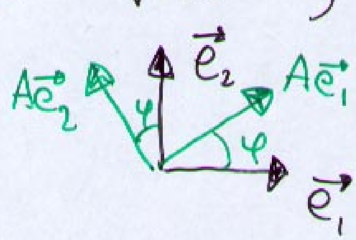
Exempel: Låt A vara avbildningen som projecerar på linjen $y=0$.



$$A\vec{e}_1 = \vec{e}_1 \quad , \quad A\vec{e}_2 = 0$$

$$\mathbb{A} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

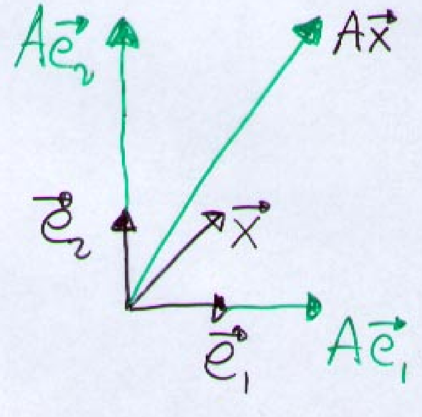
Exempel: Vridning med vinkel φ moturs



$$A\vec{e}_1 = \cos\varphi\, \vec{e}_1 + \sin\varphi\, \vec{e}_2$$
$$A\vec{e}_2 = -\sin\varphi\, \vec{e}_1 + \cos\varphi\, \vec{e}_2$$

$$A = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix}$$

Exempel: Låt A vara avbildningen som multiplicerar x-koordinaten med 2 och y-koordinaten med 3.



$$A\vec{e}_1 = 2\vec{e}_1, \quad A\vec{e}_2 = 3\vec{e}_2$$

$$A = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$$

Om A och B är två linjära avbildningar så är deras sammansättning

$$C\vec{x} = B \circ A\vec{x} = B(A\vec{x})$$

också linjär.

$$C(\vec{x}+\vec{y}) = B(A(\vec{x}+\vec{y})) =$$
$$= B(A\vec{x} + A\vec{y}) = B(A\vec{x}) + B(A\vec{y})$$

$$C(k\vec{x}) = B(A(k\vec{x})) = B(k(A\vec{x}))$$
$$= k\,B(A\vec{x}) = k\,C\vec{x}$$

$$C\vec{e}_1 = B(A\vec{e}_1) = B(a_{11}\vec{e}_1 + a_{21}\vec{e}_2)$$ 18

$$= a_{11}B\vec{e}_1 + a_{21}B\vec{e}_2 =$$

$$= a_{11}(b_{11}\vec{e}_1 + b_{21}\vec{e}_2) + a_{21}(b_{12}\vec{e}_1 + b_{22}\vec{e}_2)$$

$$= (a_{11}b_{11} + a_{21}b_{12})\vec{e}_1 + (a_{11}b_{21} + a_{21}b_{22})\vec{e}_2$$

$$C\vec{e}_2 = B(A\vec{e}_2) = B(a_{12}\vec{e}_1 + a_{22}\vec{e}_2) =$$

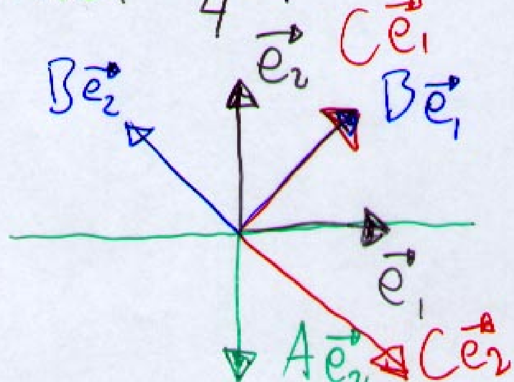
$$= a_{12}B\vec{e}_1 + a_{22}B\vec{e}_2 =$$

$$= a_{12}(b_{11}\vec{e}_1 + b_{21}\vec{e}_2) + a_{22}(b_{12}\vec{e}_1 + b_{22}\vec{e}_2)$$

$$= (a_{12}b_{11} + a_{22}b_{12})\vec{e}_1 + (a_{12}b_{21} + a_{22}b_{22})\vec{e}_2$$

$$\mathbb{C} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

Exempel: Låt A vara spegling i x-axeln och B rotation moturs med vinkel $\frac{\pi}{4}$.

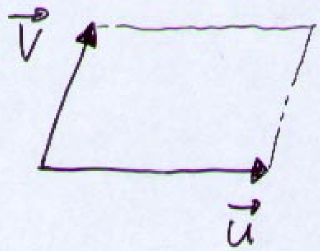


$$B = \begin{pmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

$$C = B \cdot A = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$



$\vec{u} = (x_1, x_2)$, $\vec{v} = (y_1, y_2)$

Sätt

$$\vec{u}_1 = (x_1, x_2, 0), \quad \vec{v}_1 = (y_1, y_2, 0)$$

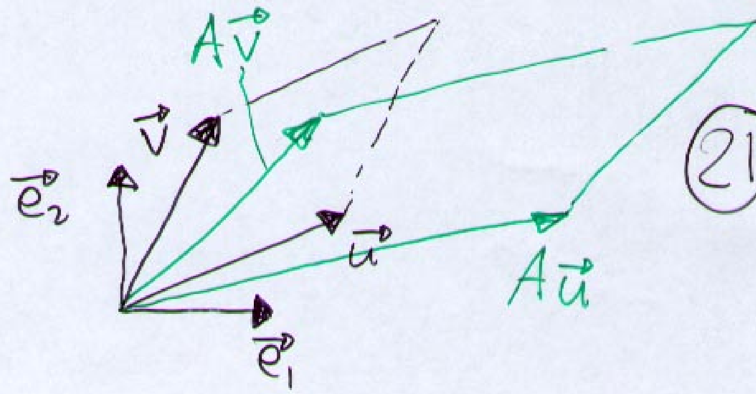
Parallellogrammets area är då

$$|(\vec{u}_1 \times \vec{v}_1)| = \left| \left(\begin{vmatrix} x_2 & 0 \\ y_2 & 0 \end{vmatrix}, -\begin{vmatrix} x_1 & 0 \\ y_1 & 0 \end{vmatrix}, \begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix} \right) \right|$$

$$= \left| \begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix} \right| = |x_1 y_2 - x_2 y_1|.$$

eller med tecken

$$\vec{u}_1 \times \vec{v}_1 = \begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix}.$$



21

$$\vec{u} = x_1 \vec{e}_1 + x_2 \vec{e}_2$$

$$\vec{v} = y_1 \vec{e}_1 + y_2 \vec{e}_2$$

$$A\vec{u} = x_1 A\vec{e}_1 + x_2 A\vec{e}_2$$

$$A\vec{v} = y_1 A\vec{e}_1 + y_2 A\vec{e}_2$$

$$A\vec{u} = (x_1 a_{11} + x_2 a_{21})\vec{e}_1 + \\ + (x_1 a_{12} + x_2 a_{22})\vec{e}_2$$

$$A\vec{v} = (y_1 a_{11} + y_2 a_{21})\vec{e}_1 \\ + (y_1 a_{12} + y_2 a_{22})\vec{e}_2$$

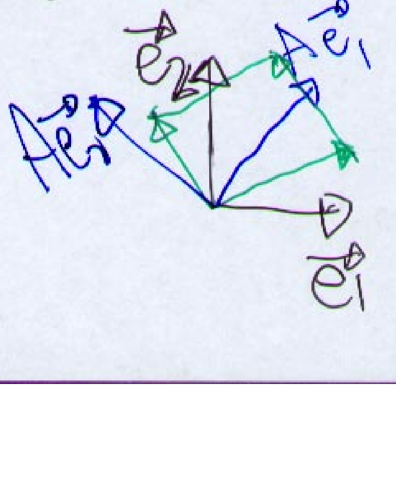
$$A\vec{u} = \left(\begin{pmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \right)^T$$

$$A\vec{v} = \left(\begin{pmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \right)^T$$

Arean av det nya parallellogrammet blir

$$\left| A \cdot \begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \end{pmatrix} \right| = \det A \begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix}.$$

Exempel: Antag att A är vridning med vinkeln φ.



Arean av parallellogrammet uppspänt av $A\vec{e}_1$ och $A\vec{e}_2$

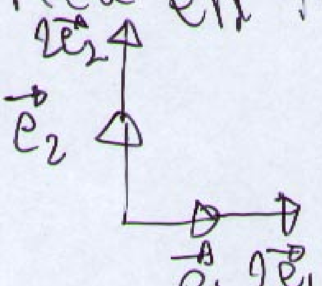
$$A = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix}$$

$$\left| A \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right| = |A| = \begin{vmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{vmatrix}$$

$$= \cos^2\varphi + \sin^2\varphi = 1$$

Så arean är 1.

Exempel 1: Låt A vara skalning med en faktor 2.



$$A = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

A ändrar areor med en faktor 4.

$$|A| = 2 \cdot 2 = 4$$