Boolesk algebra

$+$	0	1
0	0	1
1	1	1

$\cdot$	0	1
0	0	0
1	0	1

$\overline{0} = 1$

$\overline{1} = 0$

Ex: 0 = falskt, 1 = sant

$+ = \vee$ (eller) $\quad \cdot = \wedge$ (och)

$\overline{x} = \neg x$ (icke)

$F \wedge S = F \quad F \vee F = F$

$S \wedge S = S \quad F \vee S = S$

Ex: $0 = \emptyset$, $1 = X$ (hela universum)

$+ = \cup$, $\cdot = \cap$

$\overline{x} = X \setminus x$

$\{1, 2, 3, 4, 5\} \setminus \{3, 6\} = \{1, 2, 4, 5\}$

Exemplen är exempel på olika sätt att representera B_1, den booleska algebran i en variabel!

Vi bildar

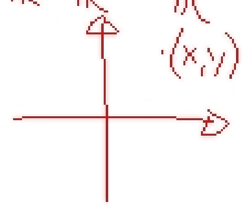
$$B_n = \underbrace{B_1 \times \dots \times B_1}_{n \text{ ssr}} = B_1^n$$

$\{0,1\} \times \{(0,0),(0,1),(1,0),(1,1)\}$

$= \{(0,(0,0)).$

$$|B_n| = 2^n$$

$\mathbb{R}^2 = \mathbb{R}^1 \times \mathbb{R}^1$



$B_1 \times B_1$

$\{0,1\} \times \{0,1\}$

$\{(0,0),(0,1),(1,0),(1,1)\}$

$$|\underbrace{B_1 \times B_1 \times \dots \times B_1}_{n \text{ ssr}}| = \underbrace{|B_1| \cdot |B_1| \cdot \ldots \cdot |B_1|}_{n \text{ ssr}} = 2^n$$

$(0,1,0)$

Man kan betrakta B_n abstrakt som en mängd med 2^n element och tre operationer $+, \cdot, \bar{\ }$ som uppfyller vissa axiom. Men det räcker att ha någon representation i minnet

FFS + SFS

$$(0,0,1) + (1,0,1) = (0+1, 0+0, 1+1)$$

$$= (1,0,1) \quad (SFS)$$

$$(0,0,1) \cdot (1,0,1) = (0 \cdot 1, 0\,0, 1 \cdot 1) =$$

$$= (0,0,1)$$

$$\overline{(0,0,1)} = (\overline{0}, \overline{0}, \overline{1}) = (1,1,0)$$

Övn. Tänk igenom följande regler i de olika modellerna.

distributiv

$$x\cdot(y+z) = (x\cdot y)+(x\cdot z)$$

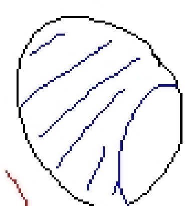
$$x+(y\cdot z) = (x+y)\cdot(x+z)$$

$0 \qquad 0$

$x+y\cdot 0 \Rightarrow x=0$ och $y=0$

$$\overline{\overline{x}} = x$$

$$\overline{X\setminus x} = X\setminus(X\setminus x) = x$$



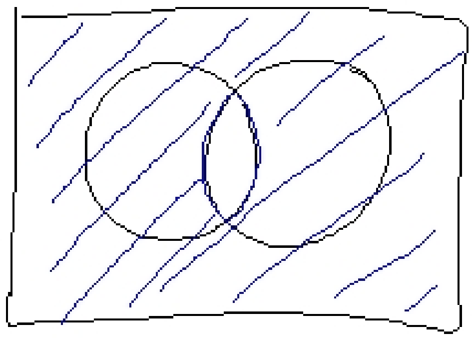
de Morgans lagar

$$\overline{x\cdot y} = \overline{x}+\overline{y}$$

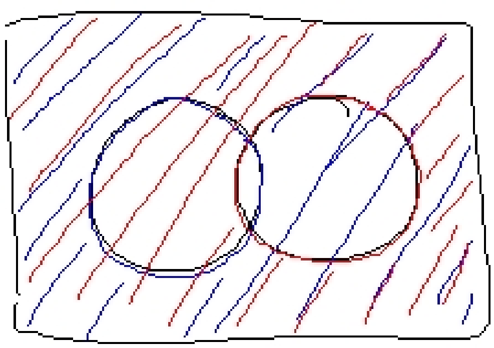
$$\overline{x+y} = \overline{x}\cdot\overline{y}$$

x	y	z	$x+(y\cdot z)$	$(x+y)\cdot(x+z)$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

$X \cup X = X$



$$\overline{X \cdot Y} = \overline{X} + \overline{Y}$$



$$X \setminus A \cap B = (X \setminus A) \cup (X \setminus B)$$

Booleska funktioner

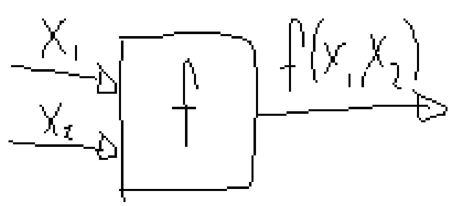
En boolesk funktion är en funktion från $f: B_n \to B_1$.

x_1	x_2	f
0	0	0
0	1	1
1	0	1
1	1	0

Sanningstabell

$$f = \overline{x}_1 x_2 + x_1 \overline{x}_2$$

disjunktiv normalform



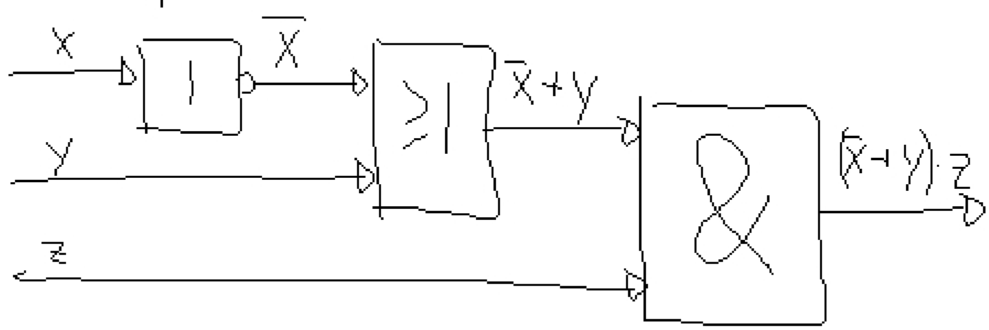
Logisk krets

Problem En funktion kan skrivas på olika sätt

$$xy + x\bar{y} = x$$

$$xyz + xy\bar{z} + x\bar{y}z + x\bar{y}\bar{z} = x$$

Ex: $(\bar{x} + y) \cdot z$



disjunktiv normal form

x	y	z	$(\overline{x}+y)\cdot z$	
0	0	0	0	
0	0	1	1	$\leftarrow \overline{x}\,\overline{y}\,z$
0	1	0	0	
0	1	1	1	$\leftarrow \overline{x}yz$
1	0	0	0	
1	0	1	0	
1	1	0	0	
1	1	1	1	$\leftarrow xyz$

$$(\overline{x}+y)\cdot z = \overline{x}\,\overline{y}\,z + \overline{x}yz + xyz$$

Vi vill ha så enkla uttryck som möjligt. För små värden på n ($n \leq 6$) kan man använda så kallade Karnaugh-diagram

x	y	f
0	0	0
0	1	1
1	0	1
1	1	1

$$f = \bar{x}y + x\bar{y} + xy$$

$$= (\bar{x}+x)\cdot y + x\bar{y} = y + x\bar{y}$$

		$\bar{y}$	y
	$x \backslash y$	0	1
$\bar{x}$	0	0	1
x	1	1	1

$$f = y + x = y + x\cdot(y+\bar{y}) = (1+x)\cdot y + x\bar{y}$$

		$\bar{y}$		y	
	$x \backslash yz$	00	01	11	10
$\bar{x}$	0	0	0	0	0
x	1	1	1	1	1
		$\bar{z}$	z	z	$\bar{z}$

$$00 \longleftrightarrow \bar{y}\bar{z}$$

$$01 \longleftrightarrow \bar{y}z$$

$$f = xyz + xy\bar{z} + x\bar{y}z + x\bar{y}\bar{z}$$

$$= x$$

$$f = \bar{x}\bar{y}\bar{z}\bar{w} + \bar{x}\bar{y}zw + \bar{x}yzw + \bar{x}yz\bar{w} + xyzw + xyz\bar{w} + x\bar{y}\bar{z}\bar{w} + x\bar{y}zw =$$

$xy \backslash zw$	00	01	11	10
00	1	0	1	0
01	0	0	1	1
11	0	0	1	1
10	1	0	1	0

$$\bar{y}\bar{z}\bar{w} + zw + yz$$

$$zw + z\bar{w} = z$$

x\yz	00	01	11	10
0	0	1	1	0
1	1	0	1	1

$$\bar{x}z + yz + x\bar{z}$$

$$= \bar{x}z + xy + x\bar{z}$$