

## Boolesk algebra

+	0	1
0	0	1
1	1	1

·	0	1
0	0	0
1	0	1

$$\overline{0} = 1$$
$$\overline{1} = 0$$

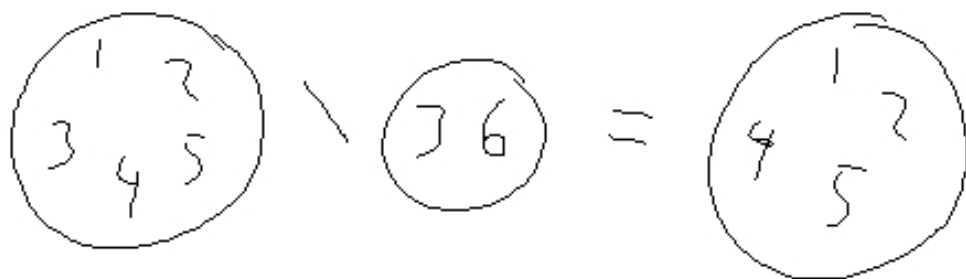
Ex: 0 = falskt, 1 = sant

+ =  $\vee$  (eller)    · =  $\wedge$  (och)

$\overline{x} = \neg x$  (icke)

$$F \wedge S = F \quad F \vee F = F$$
$$S \wedge S = S \quad F \vee S = S$$

$$\begin{aligned} \text{Ex: } 0 &= \emptyset, \quad 1 = X \text{ (hela universum)} \\ + &= \cup, \quad \cdot = \cap \\ \bar{x} &= X \setminus x \end{aligned}$$



Exemplen är exempel på olika sätt att representera  $B_1$ , den booleska algebraen i en variabel.

$V_i$  bildar

$$\underbrace{\{0,1\} \times \{(0,0), (0,1), (1,0), (1,1)\}}_{n \text{ sgr}} \quad B_n = \underbrace{B_1 \times \dots \times B_1}_{n \text{ sgr}} = B_1^n$$

$$= \{(0, (a_i))\}$$

$$|B_n| = 2^n \quad \mathbb{R}^2 = \mathbb{R}' \times \mathbb{R}' \quad B_1 \times B_1$$

$$|B_1 \times B_1 \times \dots \times B_1| =$$

$$\begin{array}{l} \uparrow \quad \rightarrow \\ \mathbb{R}' \quad \mathbb{R}' \\ (x,y) \\ \{0,1\} \times \{0,1\} \\ \{(0,0), (0,1), (1,0), (1,1)\} \end{array}$$

$$= \underbrace{|B_1| \cdot |B_1| \cdot \dots \cdot |B_1|}_{n \text{ sgr}} = 2^n$$

$$(0, 1, 0)$$

Man kan betrakta  $B_n$  abstrakt som en mängd med  $2^n$  element och tre operationer  $+$ ,  $\cdot$ ,  $-$  som uppfyller vissa axiomen. Men det räcker att ha någon representation i minnet.

Föreläsning 10, sid 5


$$\begin{aligned} & FFS + SFS \\ (0,0,1) + (1,0,1) &= (0+1, 0+0, 1+1) \\ &= (1,0,1) \quad (SFS) \\ (0,0,1) \cdot (1,0,1) &= (0 \cdot 1, 0 \cdot 0, 1 \cdot 1) = \\ &= (0,0,1) \\ \overline{(0,0,1)} &= (\bar{0}, \bar{0}, \bar{1}) = (1,1,0) \end{aligned}$$

Övn. Tänk igenom följande regler i de olika modellerna:

distributiv {  $X \cdot (Y + Z) = (X \cdot Y) + (X \cdot Z)$   
 $X + (Y \cdot Z) = (X + Y) \cdot (X + Z)$

$\overset{0}{\overline{X}} = \overset{0}{X}$   $X + Y = 0 \Rightarrow X = 0$  och  $Y = 0$

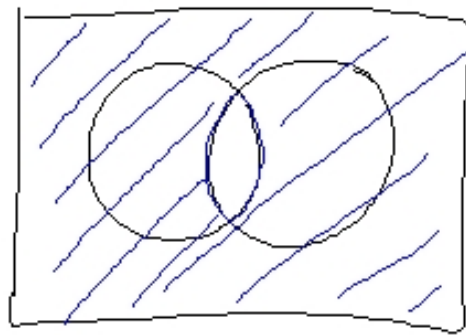
de Morgans lagar {  $\overline{X \cdot Y} = \overline{X} + \overline{Y}$   
 $\overline{X + Y} = \overline{X} \cdot \overline{Y}$

$\overline{X \cdot X} =$    
 $X \cdot (X \cdot X)$   
 $= X$

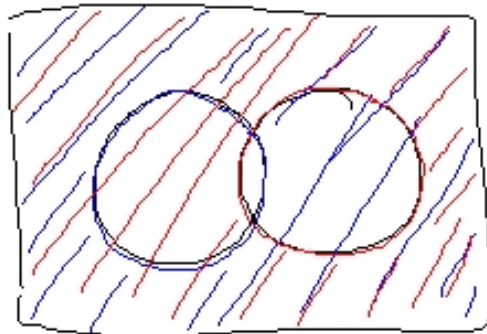
Föreläsning 10, sid 7

$x$	$y$	$z$	$x + (y \cdot z)$	$(x+y) \cdot (x+z)$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

$$X \cup X = X$$



$$\overline{X \cap Y} \\ = \bar{X} + \bar{Y}$$



$$X \setminus A \cap B \\ = (X \setminus A) \cup (X \setminus B)$$



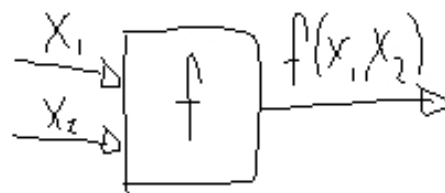
## Booleska funktioner

En booleesk funktion är en funktion från  $f: B_n \rightarrow B_1$ .

$f = \bar{X}_1 X_2 + X_1 \bar{X}_2$   
disjunktiv  
normalform

$X_1$	$X_2$	$f$
0	0	0
0	1	1
1	0	1
1	1	0

Sanningsstabell



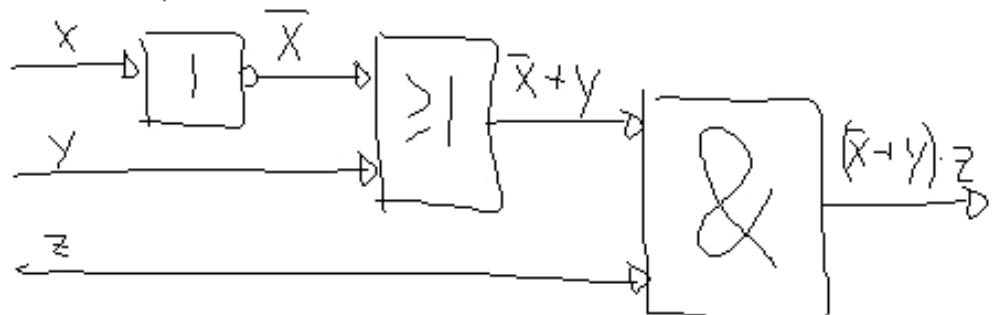
Logisk krets

Problemen En funktion kan skrivas på olika sätt

$$xy + x\bar{y} = x$$

$$xyz + xy\bar{z} + x\bar{y}z + x\bar{y}\bar{z} = x$$

Ex:  $(\bar{x} + y) \cdot z$



Disjunktiv normal form

$x$	$y$	$z$	$(\bar{x} + y) \cdot z = \bar{x}\bar{y}z + \bar{x}yz + xy\bar{z}$
0	0	0	0
0	0	1	1 ← $\bar{x}\bar{y}z$
0	1	0	0
0	1	1	1 ← $\bar{x}yz$
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1 ← $xy\bar{z}$

Vi vill ha så enkla uttryck som möjligt. För små värden på  $n$  ( $n \leq 6$ ) kan man använda så kallade Karnaugh-diagram.

$x$	$y$	$f$
0	0	0
0	1	1
1	0	1
1	1	1

$$f = \bar{x}y + x\bar{y} + xy$$

$$= (\bar{x} + x) \cdot y + x\bar{y} = y + x\bar{y}$$

$$f = y + x = y + x \cdot (y + \bar{y}) = (1+x) \cdot y + x\bar{y}$$

Föreläsning 10, sid 13

		$\bar{y}$		$y$	
		00	01	11	10
$\bar{x}$	0	0	0	0	0
$x$	1	1	1	1	1

$$00 \leftrightarrow \bar{y}\bar{z}$$

$$01 \leftrightarrow \bar{y}z$$

$$f = x y z + x y \bar{z} + x \bar{y} z + x \bar{y} \bar{z}$$

$$= x$$

$$f = \bar{x}\bar{y}\bar{z}\bar{w} + \bar{x}\bar{y}zw + \bar{x}y\bar{z}w + \bar{x}y\bar{z}\bar{w} + x\bar{y}z\bar{w} + x\bar{y}zw + xy\bar{z}\bar{w} + xy\bar{z}w =$$

		$\bar{z}$		$z$	
		00	01	11	10
$x\bar{y}$	00	1	0	1	0
	01	0	0	1	1
$x\bar{y}$	11	0	0	1	1
	10	1	0	1	0

[  $\bar{x}$  ] [  $y$  ] [  $x$  ]

[  $w$  ] [  $\bar{w}$  ]

$$\bar{y}\bar{z}\bar{w} + zw + yz$$

$$zw + z\bar{w} = z$$

Föreläsning 10, sid 15

$x \backslash yz$	00	01	11	10
0	0	1	1	0
1	1	0	1	1

Handwritten annotations on the Karnaugh map:  
 - A red bracket above the top row (01, 11) is labeled  $z$ .  
 - A red bracket below the bottom row (11, 10) is labeled  $y$ .  
 - A blue circle encloses the cells (0,01) and (0,11).  
 - A blue circle encloses the cells (1,11) and (1,10).  
 - A green circle encloses the cells (0,01) and (1,01).  
 - A green circle encloses the cells (1,11) and (1,10).  
 - A green circle encloses the cell (1,10).  
 - A red circle encloses the cell (1,11).  
 - A red circle encloses the cell (1,10).

$$\begin{aligned} & \bar{x}z + yz + x\bar{z} \quad \star \\ = & \bar{x}z + xy + x\bar{z} \quad \star \end{aligned}$$