Theory - Primal and dual Simplex method

(P) $\min\ c^T x$ s.t. $Ax = b$, $x \geq 0$

(D) $\max\ b^T y$ s.t. $A^T y + s = c$, $s \geq 0$

Partition: $A = (B \ \ N)$ (B with linearly independent columns, $x_N = 0$, $s_B = 0$)

$$\begin{pmatrix} B & N \\ 0 & I \end{pmatrix} \begin{pmatrix} x_B \\ x_N \end{pmatrix} = \begin{pmatrix} b \\ 0 \end{pmatrix} \quad (1)$$

$$\begin{pmatrix} B^T & 0 \\ N^T & I \end{pmatrix} \begin{pmatrix} y \\ s_N \end{pmatrix} = \begin{pmatrix} c_B \\ c_N \end{pmatrix} \quad (2)$$

b.f.s. if $x_B \geq 0$

b.f.s if $s_N \geq 0$

Primal Simplex

- Given $x_B \geq 0$ (b.f.s. to (P)) solve (2) to obtain y and s_N.

Some $(s_N)_t$ might be < 0

primal feasibility

$(x_N)_t$ enters the base

- For $(s_N)_t < 0$ find search direction p^t from

$$\begin{pmatrix} B & N \\ 0 & I \end{pmatrix} \begin{pmatrix} p_B \\ p_N \end{pmatrix} = \begin{pmatrix} 0 \\ e_t \end{pmatrix}$$

$c^T p^t = (s_N)_t$

- Compute maximal steplength α_{max} and limiting constraint r:

$$\alpha_{max} = \min_i \left\{ \frac{(x_B)_i}{-(p_B)_i} \,\middle|\, (p_B)_i < 0 \right\}$$

$$r = \operatorname{argmin}_i \{ \quad \text{---}\,\text{"}\,\text{---} \quad \}$$

want $x_B + \alpha p_B \geq 0$

$(x_B)_r$ leaves the base

- Take the step:

$x + \alpha_{max} p$

Dual Simplex

- Given $s_N \geq 0$ (b.f.s to (D)) solve (1) to obtain x_B.

Some $(x_B)_t$ might be < 0

dual feasibility

$(x_B)_t$ leaves the base

- For $(x_B)_t < 0$ find search directions q^t and η_N^t from

$$\begin{pmatrix} B^T & 0 \\ N^T & I \end{pmatrix} \begin{pmatrix} q \\ \eta_N \end{pmatrix} = \begin{pmatrix} -e_t \\ 0 \end{pmatrix}$$

$b^T q^t = -(x_B)_t$

- Compute maximal steplength α_{max} and limiting constraint r:

$$\alpha_{max} = \min_i \left\{ \frac{(s_N)_i}{-(\eta_N)_i} \,\middle|\, (\eta_N)_i < 0 \right\}$$

$$r = \operatorname{argmin}_i \{ \quad \text{---}\,\text{"}\,\text{---} \quad \}$$

want $s_N + \alpha \eta_N \geq 0$

$(x_N)_r$ enters the base

- Take the step:

$y + \alpha_{max} q$

$s_B + \alpha_{max} e_t$

$s_N + \alpha_{max} \eta_N$

Optimal when we have both primal and dual feasibility, since complementarity $x_j s_j = 0 \ \forall j$ is always fulfilled.