KTH Mathematics

SF2822 Applied nonlinear optimization

Barrier methods for linear and semidefinite programming

Linear programming	**Semidefinite programming**
Given $A \in I\!R^{m\times n}$, $b \in I\!R^m$ and $c \in I\!R^n$.	Given $B \in I\!R^{m\times m}$, B symmetric, $A_j \in I\!R^{m\times m}$, A_j symmetric, for $j = 1,\dots,n$ and $c \in I\!R^n$.
Let $e = (1\ 1\ \dots 1)^T$, $g(x) = Ax - b$ and $G(x) = \operatorname{diag}(g(x))$.	Let $G(x) = \sum_{j=1}^n A_j x_j - B$.
Primal problem: $(P) \quad \begin{array}{ll} \min & c^T x \\ \text{då} & Ax \ge b. \end{array}$	Primal problem: $(P) \quad \begin{array}{ll} \min & c^T x \\ \text{då} & \sum_{j=1}^n A_j x_j \succeq B. \end{array}$
Dual problem: $(D) \quad \begin{array}{ll} \max & b^T y \\ \text{då} & A^T y = c, \\ & y \ge 0. \end{array}$	Dual problem: $(D) \quad \begin{array}{ll} \max & \operatorname{trace}(BY) \\ \text{då} & \operatorname{trace}(A_j Y) = c_j, \quad j = 1,\dots,n, \\ & Y = Y^T \succeq 0. \end{array}$
Duality gap: $c^T x - b^T y = g(x)^T y.$	Duality gap: $c^T x - \operatorname{trace}(BY) = \operatorname{trace}(G(x)Y).$
Barrier-transformed problem: $(P_\mu) \quad \min c^T x - \mu \sum_{i=1}^m \ln(g_i(x)).$	Barrier-transformed problem: $(P_\mu) \quad \min c^T x - \mu \ln(\det(G(x))).$
Optimality conditions: $c - \mu A^T G(x)^{-1} e = 0.$	Optimality conditions: $c_j - \mu \operatorname{trace}(A_j G(x)^{-1}) = 0, \quad j = 1,\dots,n.$
With $y = \mu G(x)^{-1} e$ the optimality conditions may be written as $\begin{aligned} c - A^T y &= 0, \\ G(x)y - \mu e &= 0. \end{aligned}$ ($g(x) > 0$, $y > 0$ implicitly)	With $Y = \mu G(x)^{-1}$ the optimality conditions may be written as $\begin{aligned} c_j - \operatorname{trace}(A_j Y) &= 0, \quad j = 1,\dots,n, \\ G(x)Y - \mu I &= 0. \end{aligned}$ ($G(x) \succ 0$, $Y \succ 0$ implicitly)
We have n primal variables and m dual variables.	We have n primal variables and $\frac{m(m+1)}{2}$ dual variables.

All results in the left columns may be obtained as a special case of the results of the right column by letting the matrices be diagonal.

It holds that

$$\frac{\partial \ln(\det(G(x)))}{\partial x_j} = \operatorname{trace}(A_j G(x)^{-1}) \quad \text{för} \quad j = 1,\dots,n.$$