

INFERENCE RULES FOR SENTENTIAL LOGIC

**Rule of Assumptions:**  
 $j (j) p$  Assumption  
 where  $p$  may be any formula.

**Rule of &E:**  
 $a_1, \dots, a_n (j) p \ \& \ q$   
 $\vdots$   
 $a_1, \dots, a_n (k) p \ \text{(or } q)$   $j \ \&E$

**Rule of &I:**  
 $a_1, \dots, a_n (j) p$   
 $\vdots$   
 $b_1, \dots, b_u (k) q$   
 $a_1, \dots, a_n, b_1, \dots, b_u (m) p \ \& \ q$   $j, k \ \&I$

**Rule of  $\neg$ E:**  
 $a_1, \dots, a_n (j) p \ \neg \ q$   
 $\vdots$   
 $b_1, \dots, b_u (k) p$   
 $a_1, \dots, a_n, b_1, \dots, b_u (m) q$   $j, k \ \neg E$

**Rule of  $\neg$ I:**  
 $j (j) p$  Assumption  
 $\vdots$   
 $a_1, \dots, a_n (k) q$   
 $\{a_1, \dots, a_n\}/j (m) p \ \neg \ q$   $j, k \ \neg I$

**Rule of  $\sim$ E:**  
 $a_1, \dots, a_n (j) \sim q$   
 $\vdots$   
 $b_1, \dots, b_u (k) q$   
 $a_1, \dots, a_n, b_1, \dots, b_u (m) \wedge$   $j, k \ \sim E$

**Rule of  $\sim$ I:**  
 $j (j) p$  Assumption  
 $\vdots$   
 $a_1, \dots, a_n (k) \wedge$   
 $\{a_1, \dots, a_n\}/j (m) \sim p$   $j, k \ \sim I$

**Rule of DN:**  
 $a_1, \dots, a_n (j) \sim \sim p$   
 $\vdots$   
 $a_1, \dots, a_n (k) p$   $j \ \text{DN}$

**Rule of  $\vee$ E:**  
 $a_1, \dots, a_n (g) p \ \vee \ q$   
 $\vdots$   
 $h (h) p$  Assumption  
 $\vdots$   
 $b_1, \dots, b_u (i) r$   
 $\vdots$   
 $j (j) q$  Assumption  
 $\vdots$   
 $c_1, \dots, c_w (k) r$   
 $X (m) r$   $g, h, i, j, k \ \vee E$   
 where  $X = \{a_1, \dots, a_n\} \cup \{b_1, \dots, b_u\}/h \cup \{c_1, \dots, c_w\}/j$ .

**Rule of  $\vee$ I:**  
 $a_1, \dots, a_n (j) p$   
 $\vdots$   
 $a_1, \dots, a_n (k) p \ \vee \ q$   $j \ \vee I$   
 or  
 $a_1, \dots, a_n (k) q \ \vee \ p$   $j \ \vee I$

**Rule of Df:**  
 If ' $(p \rightarrow q) \ \& \ (q \rightarrow p)$ ' occurs as the entire formula at a line  $j$ , then at line  $k$  we may write ' $p \rightarrow q$ ', labeling the line ' $j$ , Df' and writing on its left the numbers from the left of  $j$ . Conversely, if ' $p \rightarrow q$ ' occurs as the entire formula at a line  $j$ , then at line  $k$  we may write ' $(p \rightarrow q) \ \& \ (q \rightarrow p)$ ', labeling the line ' $j$ , Df' and writing on its left the numbers from the left of  $j$ .

**Rule of EFQ:**  
 $a_1, \dots, a_n (j) \wedge$   
 $\vdots$   
 $a_1, \dots, a_n (k) p$   $j \ \text{EFQ}$

Som alternativ till Df är det tillåtet att använda

**Rule of  $\leftrightarrow$ E:**  
 $a_1, \dots, a_m (j) p \leftrightarrow q$   
 (or  $a_1, \dots, a_m (j) q \leftrightarrow p$ )  
 $\vdots$   
 $b_1, \dots, b_n (k) p$   
 $\vdots$   
 $a_1, \dots, a_m, b_1, \dots, b_n (l) q$   $j, k \ \leftrightarrow E$

**Rule of  $\leftrightarrow$ I:**  
 $g (g) p$  Assumption  
 $\vdots$   
 $a_1, \dots, a_m (h) q$   
 $\vdots$   
 $j (j) q$  Assumption  
 $\vdots$   
 $b_1, \dots, b_n (k) p$   
 $\vdots$   
 $X (l) p \leftrightarrow q$   $g, h, j, k \ \leftrightarrow I$   
 where  $X = \{a_1, \dots, a_m\}/g \cup \{b_1, \dots, b_n\}/j$

INFERENCE RULES FOR FIRST-ORDER LOGIC

**Rule of  $\forall$ E:**  
 $a_1, \dots, a_n (j) (\forall v)\phi v$   
 $\vdots$   
 $a_1, \dots, a_n (k) \phi t$   $j \ \forall E$   
 where  $\phi t$  is obtained from  $\phi v$  by replacing every occurrence of  $v$  in  $\phi v$  with  $t$ .

**Rule of  $\forall$ I:**  
 $a_1, \dots, a_n (j) \phi t$   
 $\vdots$   
 $a_1, \dots, a_n (k) (\forall v)\phi v$   $j \ \forall I$   
 where  $t$  is not in any of the formulae on lines  $a_1, \dots, a_n$  and  $\phi v$  is obtained from  $\phi t$  by replacing every occurrence of  $t$  in  $\phi t$  with  $v$ ,  $v$  a variable not already in  $\phi t$ .

**Rule of  $\exists$ I:**  
 $a_1, \dots, a_n (j) \phi t$   
 $\vdots$   
 $a_1, \dots, a_n (k) (\exists v)\phi v$   $j \ \exists I$   
 where  $\phi v$  is obtained from  $\phi t$  by replacing at least one occurrence of  $t$  in  $\phi t$  with  $v$ ,  $v$  a variable not already in  $\phi t$ .

**Rule of  $\exists$ E:**  
 $a_1, \dots, a_n (i) (\exists v)\phi v$   
 $\vdots$   
 $j (j) \phi t$  Assumption  
 $\vdots$   
 $b_1, \dots, b_u (k) \psi$   
 $\vdots$   
 $X (m) \psi$   $i, j, k \ \exists E$   
 where  $X = \{a_1, \dots, a_n\} \cup \{b_1, \dots, b_u\}/j$  and  $t$  is not in (i) ' $(\exists v)\phi v$ ', (ii)  $\psi$ , or (iii) any of the formulae at lines  $b_1, \dots, b_u$  other than  $j$ .

**Rule of =E:**  
 $a_1, \dots, a_n (j) t_1 = t_2$   
 $\vdots$   
 $b_1, \dots, b_u (k) \phi t_1$   
 $\vdots$   
 $a_1, \dots, a_n, b_1, \dots, b_u (m) \phi t_2$   $j, k \ =E$   
 where  $\phi t_2$  is obtained from  $\phi t_1$  by replacing at least one occurrence of  $t_1$  in  $\phi t_1$  with  $t_2$ .

**Rule of =I:**  
 $(j) t = t$  =I  
 where  $t$  is any individual constant.

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**Rule of Sequent Introduction:** Suppose the sequent  $r_1, \dots, r_n \vdash_{NK} s$  is a substitution-instance of the sequent  $p_1, \dots, p_n \vdash_{NK} q$ , that we have already proved the sequent  $p_1, \dots, p_n \vdash_{NK} q$ , and that the formulae  $r_1, \dots, r_n$  occur at lines  $j_1, \dots, j_n$  in a proof. Then we may infer  $s$  at line  $k$ , labeling the line ' $j_1, \dots, j_n$  SI (Identifier)' and writing on the left all the numbers which occur on the left of lines  $j_1, \dots, j_n$ . As a special case, when  $n = 0$  and  $\vdash_{NK} s$  is a substitution-instance of some theorem  $\vdash_{NK} q$  which we have already proved, we may introduce a new line  $k$  into a proof with the formula  $s$  at it and no numbers on the left, labeling the line 'TI (Identifier)'.

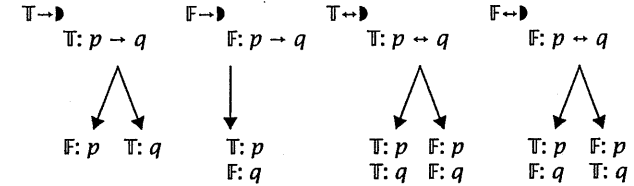
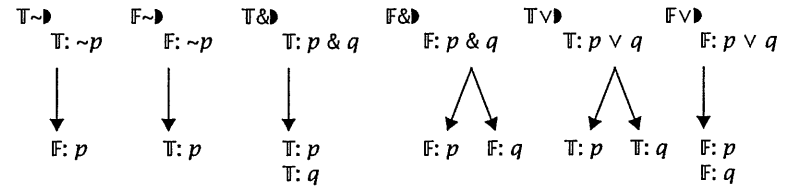
- (a)  $A \vee B, \sim A \vdash_{NK} B$ ; or:  $A \vee B, \sim B \vdash_{NK} A$  (DS)
- (b)  $A \rightarrow B, \sim B \vdash_{NK} \sim A$  (MT)
- (c)  $A \vdash_{NK} B \rightarrow A$  (PMI)
- (d)  $\sim A \vdash_{NK} A \rightarrow B$  (PMI)
- (e)  $A \vdash_{NK} \sim \sim A$  (DN\*)
- (f)  $\sim(A \& B) \dashv\vdash_{NK} \sim A \vee \sim B$  (DeM)
- (g)  $\sim(A \vee B) \dashv\vdash_{NK} \sim A \& \sim B$  (DeM)
- (h)  $\sim(\sim A \vee \sim B) \dashv\vdash_{NK} A \& B$  (DeM)
- (i)  $\sim(\sim A \& \sim B) \dashv\vdash_{NK} A \vee B$  (DeM)
- (j)  $A \rightarrow B \dashv\vdash_{NK} \sim A \vee B$  (Imp)
- (k)  $\sim(A \rightarrow B) \dashv\vdash_{NK} A \& \sim B$  (Neg-Imp)
- (l)  $A * B \vdash_{NK} B * A$  (Com)
- (m)  $A \& (B \vee C) \dashv\vdash_{NK} (A \& B) \vee (A \& C)$  (Dist)
- (n)  $A \vee (B \& C) \dashv\vdash_{NK} (A \vee B) \& (A \vee C)$  (Dist)
- (p)  $\vdash_{NK} A \vee \sim A$  (LEM)
- (q)  $A * B \dashv\vdash_{NK} \sim \sim A * \sim \sim B$ ; or:  $\sim \sim A * B$ ; or:  $A * \sim \sim B$  (SDN)
- (r)  $\sim(A * B) \dashv\vdash_{NK} \sim(\sim \sim A * \sim \sim B)$ ; or:  $\sim(\sim \sim A * B)$ ; or:  $\sim(A * \sim \sim B)$  (SDN)

' $p \dashv\vdash_{NK} q$ ' står här för ' $p \vdash_{NK} q$  och  $q \vdash_{NK} p$ '.  
 I (l) står '\*' för '&', '∨' eller '↔'.  
 I (q) och (r) står '\*' för '&', '∨', '→' eller '↔'.

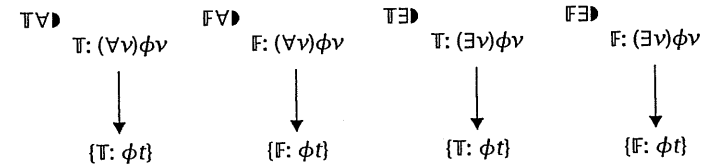
**Extension (1) to the Rule of Sequent Introduction:** If the formula at a line  $j$  in a proof has any of the forms ' $\sim(\forall v)\phi v$ ', ' $(\exists v)\sim\phi v$ ', ' $\sim(\exists v)\phi v$ ' or ' $(\forall v)\sim\phi v$ ', then at line  $k$  we may infer the provably equivalent formula of the form ' $(\exists v)\sim\phi v$ ', ' $\sim(\forall v)\phi v$ ', ' $(\forall v)\sim\phi v$ ' or ' $\sim(\exists v)\phi v$ ' respectively, labeling the line ' $j$  SI (QS)' and writing on the left the same numbers as occur on the left of line  $j$ .

**Extension (2) to the Rule of Sequent Introduction:** For any closed sentence  $\phi v$ , if  $\phi v$  has been inferred at a line  $j$  in a proof and  $\phi v'$  is a single-variable alphabetic variant of  $\phi v$ , then at line  $k$  we may write  $\phi v'$ , labeling the line ' $j$  SI (AV)' and carrying down on its left the same numbers as are on the left of line  $j$ .

TABLÅREGLER I SATSLOGIK



TABLÅREGLER I PREDIKATLOGIK



PEANOS AXIOM

**Språk** (tolkningen i standardmodellen inom [ ]):

- en 0-ställig funktionssymbol (dvs en individkonstant) 0 [talet 0]
- en 1-ställig funktionssymbol  $S$  [nästa tal]
- två 2-ställiga funktionssymboler + och \* [addition och multiplikation]
- (vi skriver t.ex.  $x + y$  och  $x * y$  i stället för  $+(x, y)$  och  $*(x, y)$ .)

**Axiom:**

- P1  $\forall x \forall y (S(x) = S(y) \rightarrow x = y)$  olika tal har olika efterföljare
  - P2  $\forall x S(x) \neq 0$  0 är inte efterföljare
  - P3  $\forall x x + 0 = x$  rekursiv definition av +, bas
  - P4  $\forall x \forall y x + S(y) = S(x + y)$  rekursiv definition av +, steg
  - P5  $\forall x x * 0 = 0$  rekursiv definition av \*, bas
  - P6  $\forall x \forall y x * S(y) = (x * y) + x$  rekursiv definition av \*, steg
  - P7  $\forall z_1 \dots \forall z_n ((\phi 0 \& \forall x (\phi x \rightarrow \phi S(x))) \rightarrow \forall x \phi x)$  induktionsaxiom
- I P7 är  $\phi x$  en godtycklig formel med alla fria variabler bland  $x, z_1, \dots, z_n$ . P7 är alltså egentligen ett oändligt antal axiom, ett s.k. **axiomschema**.