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INFERENCE RULES FOR SENTENTIAL LOGIC

Rule of Assumptions:

j (j) p Assumption

where p may be any formula.

Rule of &E:

$a_1,\ldots,a_n$ (j) $p \,\&\, q$
⋮
$a_1,\ldots,a_n$ (k) p (or q) j &E

Rule of &I:

$a_1,\ldots,a_n$ (j) p
⋮
$b_1,\ldots,b_u$ (k) q
⋮
$a_1,\ldots,a_n,b_1,\ldots,b_u$ (m) $p \,\&\, q$ j,k &I

Rule of →E:

$a_1,\ldots,a_n$ (j) $p \to q$
⋮
$b_1,\ldots,b_u$ (k) p
⋮
$a_1,\ldots,a_n,b_1,\ldots,b_u$ (m) q j,k →E

Rule of →I:

j (j) p Assumption
⋮
$a_1,\ldots,a_n$ (k) q
⋮
$\{a_1,\ldots,a_n\}$/j (m) $p \to q$ j,k →I

Rule of ~E:

$a_1,\ldots,a_n$ (j) $\sim q$
⋮
$b_1,\ldots,b_u$ (k) q
⋮
$a_1,\ldots,a_n,b_1,\ldots,b_u$ (m) $\curlywedge$ j,k ~E

Rule of ~I:

j (j) p Assumption
⋮
$a_1,\ldots,a_n$ (k) $\curlywedge$
⋮
$\{a_1,\ldots,a_n\}$/j (m) $\sim p$ j,k ~I

Rule of DN:

$a_1,\ldots,a_n$ (j) $\sim\sim p$
⋮
$a_1,\ldots,a_n$ (k) p j DN

Rule of ∨E:

$a_1,\ldots,a_n$ (g) $p \vee q$
⋮
h (h) p Assumption
⋮
$b_1,\ldots,b_u$ (i) r
⋮
j (j) q Assumption
⋮
$c_1,\ldots,c_w$ (k) r
⋮
X (m) r g,h,i,j,k ∨E

where $X = \{a_1,\ldots,a_n\} \cup \{b_1,\ldots,b_u\}/h \cup \{c_1,\ldots,c_w\}/j$.

Rule of ∨I:

$a_1,\ldots,a_n$ (j) p
⋮
$a_1,\ldots,a_n$ (k) $p \vee q$ j ∨I
or
$a_1,\ldots,a_n$ (k) $q \vee p$ j ∨I

Rule of Df:

If ⌜$(p \to q) \,\&\, (q \to p)$⌝ occurs as the entire formula at a line j, then at line k we may write ⌜$p \leftrightarrow q$⌝, labeling the line 'j, Df' and writing on its left the numbers from the left of j. Conversely, if ⌜$p \leftrightarrow q$⌝ occurs as the entire formula at a line j, then at line k we may write ⌜$(p \to q) \,\&\, (q \to p)$⌝, labeling the line 'j, Df' and writing on its left the numbers from the left of j.

Rule of EFQ:

$a_1,\ldots,a_n$ (j) $\curlywedge$
⋮
$a_1,\ldots,a_n$ (k) p j EFQ

Som alternativ till *Df* är det tillåtet att använda

Rule of ↔E:

$a_1,\ldots,a_m$ (j) $p \leftrightarrow q$
(or $a_1,\ldots,a_m$ (j) $q \leftrightarrow p$)
⋮
$b_1,\ldots,b_n$ (k) p
⋮
$a_1,\ldots,a_m,b_1,\ldots,b_n$ (l) q j,k ↔E

Rule of ↔I:

g (g) p Assumption
⋮
$a_1,\ldots,a_m$ (h) q
⋮
j (j) q Assumption
⋮
$b_1,\ldots,b_n$ (k) p
⋮
X (l) $p \leftrightarrow q$ g,h,j,k ↔I

where $X = \{a_1,\ldots,a_m\}/g \cup \{b_1,\ldots,b_n\}/j$

INFERENCE RULES FOR FIRST-ORDER LOGIC

Rule of ∀E:

$a_1,\ldots,a_n$ (j) $(\forall v)\phi v$
⋮
$a_1,\ldots,a_n$ (k) ϕt j ∀E

where ϕt is obtained from ϕv by replacing *every* occurrence of v in ϕv with t.

Rule of ∀I:

$a_1,\ldots,a_n$ (j) ϕt
⋮
$a_1,\ldots,a_n$ (k) $(\forall v)\phi v$ j ∀I

where t is not in any of the formulae on lines $a_1,\ldots,a_n$ and ϕv is obtained from ϕt by replacing *every* occurrence of t in ϕt with v, v a variable not already in ϕt.

Rule of ∃E:

$a_1,\ldots,a_n$ (i) $(\exists v)\phi v$
⋮
j (j) ϕt Assumption
⋮
$b_1,\ldots,b_u$ (k) ψ
⋮
X (m) ψ i,j,k ∃E

where $X = \{a_1,\ldots,a_n\} \cup \{b_1,\ldots,b_u\}/j$ and t is not in (i) ⌜$(\exists v)\phi v$⌝, (ii) ψ, or (iii) any of the formulae at lines $b_1,\ldots,b_u$ other than j.

Rule of ∃I:

$a_1,\ldots,a_n$ (j) ϕt
⋮
$a_1,\ldots,a_n$ (k) $(\exists v)\phi v$ j ∃I

where ϕv is obtained from ϕt by replacing *at least one* occurrence of t in ϕt with v, v a variable not already in ϕt.

Rule of =E:

$a_1,\ldots,a_n$ (j) $t_1 = t_2$
⋮
$b_1,\ldots,b_u$ (k) ϕt_1
⋮
$a_1,\ldots,a_n,b_1,\ldots,b_u$ (m) ϕt_2 j,k =E

where ϕt_2 is obtained from ϕt_1 by replacing *at least one* occurrence of t_1 in ϕt_1 with t_2.

Rule of =I:

(j) $t = t$ =I

where t is any individual constant.

HELA DENNA SIDA ÄR SI-REGLER

Rule of Sequent Introduction: Suppose the sequent $r_1,\ldots,r_n \vdash_{NK} s$ is a substitution-instance of the sequent $p_1,\ldots,p_n \vdash_{NK} q$, that we have already proved the sequent $p_1,\ldots,p_n \vdash_{NK} q$, and that the formulae $r_1,\ldots,r_n$ occur at lines $j_1,\ldots,j_n$ in a proof. Then we may infer s at line k, labeling the line '$j_1,\ldots,j_n$ SI ⟨Identifier⟩' and writing on the left all the numbers which occur on the left of lines $j_1,\ldots,j_n$. As a special case, when $n = 0$ and $\vdash_{NK} s$ is a substitution-instance of some theorem $\vdash_{NK} q$ which we have already proved, we may introduce a new line k into a proof with the formula s at it and no numbers on the left, labeling the line 'TI ⟨Identifier⟩'.

(a)	$A \vee B, {\sim}A \vdash_{NK} B$; or: $A \vee B, {\sim}B \vdash_{NK} A$	(DS)
(b)	$A \rightarrow B, {\sim}B \vdash_{NK} {\sim}A$	(MT)
(c)	$A \vdash_{NK} B \rightarrow A$	(PMI)
(d)	${\sim}A \vdash_{NK} A \rightarrow B$	(PMI)
(e)	$A \vdash_{NK} {\sim}{\sim}A$	(DN+)
(f)	${\sim}(A \,\&\, B) \dashv\vdash_{NK} {\sim}A \vee {\sim}B$	(DeM)
(g)	${\sim}(A \vee B) \dashv\vdash_{NK} {\sim}A \,\&\, {\sim}B$	(DeM)
(h)	${\sim}({\sim}A \vee {\sim}B) \dashv\vdash_{NK} A \,\&\, B$	(DeM)
(i)	${\sim}({\sim}A \,\&\, {\sim}B) \dashv\vdash_{NK} A \vee B$	(DeM)
(j)	$A \rightarrow B \dashv\vdash_{NK} {\sim}A \vee B$	(Imp)
(k)	${\sim}(A \rightarrow B) \dashv\vdash_{NK} A \,\&\, {\sim}B$	(Neg-Imp)
(l)	$A * B \vdash_{NK} B * A$	(Com)
(m)	$A \,\&\, (B \vee C) \dashv\vdash_{NK} (A \,\&\, B) \vee (A \,\&\, C)$	(Dist)
(n)	$A \vee (B \,\&\, C) \dashv\vdash_{NK} (A \vee B) \,\&\, (A \vee C)$	(Dist)
(p)	$\vdash_{NK} A \vee {\sim}A$	(LEM)
(q)	$A * B \dashv\vdash_{NK} {\sim}{\sim}A * {\sim}{\sim}B$; or: ${\sim}{\sim}A * B$; or: $A * {\sim}{\sim}B$	(SDN)
(r)	${\sim}(A * B) \dashv\vdash_{NK} {\sim}({\sim}{\sim}A * {\sim}{\sim}B)$; or: ${\sim}({\sim}{\sim}A * B)$; or: ${\sim}(A * {\sim}{\sim}B)$	(SDN)

'$p \dashv\vdash_{NK} q$' står här för '$p \vdash_{NK} q$ och $q \vdash_{NK} p$'.
I (l) står '$*$' för '$\&$', '$\vee$' eller '$\leftrightarrow$'.
I (q) och (r) står '$*$' för '$\&$', '$\vee$', '$\rightarrow$' eller '$\leftrightarrow$'.

Extension (1) to the Rule of Sequent Introduction: If the formula at a line j in a proof has any of the forms ⌜${\sim}(\forall v)\phi v$⌝, ⌜$(\exists v){\sim}\phi v$⌝, ⌜${\sim}(\exists v)\phi v$⌝ or ⌜$(\forall v){\sim}\phi v$⌝, then at line k we may infer the provably equivalent formula of the form ⌜$(\exists v){\sim}\phi v$⌝, ⌜${\sim}(\forall v)\phi v$⌝, ⌜$(\forall v){\sim}\phi v$⌝ or ⌜${\sim}(\exists v)\phi v$⌝ respectively, labeling the line 'j SI (QS)' and writing on the left the same numbers as occur on the left of line j.

Extension (2) to the Rule of Sequent Introduction: For any closed sentence ϕv, if ϕv has been inferred at a line j in a proof and $\phi v'$ is a single-variable alphabetic variant of ϕv, then at line k we may write $\phi v'$, labeling the line 'j SI (AV)' and carrying down on its left the same numbers as are on the left of line j.

TABLÅREGLER I SATSLOGIK

𝕋~▸	𝕋: ~p	𝔽: p
𝔽~▸	𝔽: ~p	𝕋: p
𝕋&▸	𝕋: p & q	𝕋: p, 𝕋: q
𝔽&▸	𝔽: p & q	𝔽: p | 𝔽: q (förgrening)
𝕋∨▸	𝕋: p ∨ q	𝕋: p | 𝕋: q (förgrening)
𝔽∨▸	𝔽: p ∨ q	𝔽: p, 𝔽: q
𝕋→▸	𝕋: p → q	𝔽: p | 𝕋: q (förgrening)
𝔽→▸	𝔽: p → q	𝕋: p, 𝔽: q
𝕋↔▸	𝕋: p ↔ q	𝕋: p, 𝕋: q | 𝔽: p, 𝔽: q (förgrening)
𝔽↔▸	𝔽: p ↔ q	𝕋: p, 𝔽: q | 𝔽: p, 𝕋: q (förgrening)

TABLÅREGLER I PREDIKATLOGIK

𝕋∀▸	𝕋: $(\forall v)\phi v$	{𝕋: ϕt}
𝔽∀▸	𝔽: $(\forall v)\phi v$	{𝔽: ϕt}
𝕋∃▸	𝕋: $(\exists v)\phi v$	{𝕋: ϕt}
𝔽∃▸	𝔽: $(\exists v)\phi v$	{𝔽: ϕt}

PEANOS AXIOM

Språk (tolkningen i standardmodellen inom [ ]):
en 0-ställig funktionssymbol (dvs en individkonstant) 0 [talet 0]
en 1-ställig funktionssymbol S [nästa tal]
två 2-ställiga funktionssymboler $+$ och $*$ [addition och multiplikation]
(vi skriver t.ex. $x+y$ och $x*y$ i stället för $+(x,y)$ och $*(x,y)$.)

Axiom:

P1	$\forall x \forall y\,(S(x) = S(y) \rightarrow x = y)$	olika tal har olika efterföljare
P2	$\forall x\, S(x) \neq 0$	0 är inte efterföljare
P3	$\forall x\, x + 0 = x$	rekursiv definition av $+$, bas
P4	$\forall x \forall y\, x + S(y) = S(x+y)$	rekursiv definition av $+$, steg
P5	$\forall x\, x * 0 = 0$	rekursiv definition av $*$, bas
P6	$\forall x \forall y\, x * S(y) = (x*y) + x$	rekursiv definition av $*$, steg
P7	$\forall z_1 \ldots \forall z_n((\phi 0 \,\&\, \forall x\,(\phi x \rightarrow \phi S(x))) \rightarrow \forall x\, \phi x)$	induktionsaxiom

I P7 är ϕx en godtycklig formel med alla fria variabler bland $x, z_1, \ldots, z_n$. P7 är alltså egentligen ett oändligt antal axiom, ett s.k. **axiomschema**.