Integration Theory / Mathematical Analysis

(5B1479/MA429)

Final Exam

You may use Friedman's book and class notes (no other books, please). Explain everything, mentioning appropriate theorems (from class or Friedman's book). **Good luck!**

1. Find the limit

$$\lim_{n\to\infty} \int_1^\infty \frac{n\sin(x/n)}{x^3} dx.$$

2. Show that is $f \in L^1(\mathbb{R}, m)$, where m denotes the one-dimensional Lebesgue measure, the function g defined by $g(x, y) = f(x + y)$ is Lebesgue integrabel on $[a, b] \times [a, b]$ for any $-\infty < a, b < \infty$.

3. Let $(X, \mathfrak{A}, \mu)$ be a measure space, and let $\{E_i\}_{i=1}^\infty$ be a sequence of measurable sets. Show that $E = \{x : x$ belongs to exactly two sets$\}$ is measurable.

4. Suppose that function f is bounded and Lebesgue integrable on $\mathbb{R}$. Show that function

$$g(t) = \int_{\mathbb{R}} \frac{f(x)}{x^2 + t^2} dx$$

is continuous and bounded on $[1, \infty]$.

5. We say that function $G(x)$ is Lipschitz if there is a constant M such that for any $|G(x) - G(y)| < M|x - y|$ for any x and y. Show that if function $F(x)$ is absolutely continuous on a closed interval $[a, b]$ then then function $G(F(x))$ is also absolutely continuous on $[a, b]$.