

# COMPUTATIONAL ALGEBRAIC GEOMETRY II

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## HOMEWORK 5

**Stabilizer groups of initial ideals.** The generic initial ideal is a Borel-fixed ideal, which means that the Borel subgroup of the general linear group stabilizes the ideal. However, the stabilizer subgroup may be larger than the Borel group.

Let  $S = \mathbb{F}_5[x_0, x_1, x_2, x_3]$  and let  $I$  be the ideal generated by the  $4 \times 4$ -pfaffians of a random skew-symmetric  $5 \times 5$ -matrix of linear forms.

- (1) Write a program that finds the five most common initial ideals after a random change random of variables.
- (2) Which of these initial ideals are Borel?
- (3) Determine the stabilizer subgroup of each of these initial ideals.

If you succeed in these tasks, replace the ideal by the ideal of  $4 \times 4$ -pfaffians of a random skew-symmetric  $5 \times 5$ -matrix of quadratic forms and repeat the study.

**Borel-fixed ideals with Hilbert polynomial  $4t + 1$ .** According to the paper by Reeves, there are twelve Borel-fixed ideals with Hilbert polynomial  $4t + 1$ . Write a program to find these twelve monomial ideals.