Estimation of the smoothing parameters in the HPMV filter

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Abstract

We suggest an optimality criterium for choosing the best smoothing parameters for the so-called Hodrick-Prescott Multivariate (HPMV) filter. We show that this criterium admits a whole set of optimal smoothing parameters, to which belong the widely used noise-to-signal ratios. We also propose explicit consistent estimators of these noise-to-signal ratios.

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1 Introduction

To estimate unobserved economic variables, three approaches are usually advocated. These include structural methods which rely on economic modeling, filtering methods which apply statistical devices and methods which combine both and to which belongs the so-called Hodrick-Prescott Multivariate (HPMV) filter, suggseted in Laxton and Teltow (1992). The HPMV filter is the standard univariate Hodrick-Prescott (HP) filter augmented with relevant economic information (see Laxton and Teltow (1992), Richardson *et al.* (2000), Boone (2000) and Chagny and Lemoine (2003), and the large lists of references therein). This is done by minimizing the residuals of one or more economic relationships involving the unobserved variables. The intuition behind this contruction is to produce an estimate of the unobserved variables which maximizes the fit to the estimated economic relationships whilest conforming to the standard properties of the HP filter. More specifically, the HPMV filter seeks to estimate the unobserved variable ($y_t,\ \ t = 1, \ldots, T$) which minimizes

$$\sum_{t=1}^{T}(x_t - y_t)^2 + \alpha_1 \sum_{t=1}^{T-2}(y_{t+2} - 2y_{t+1} + y_t)^2 + \alpha_2 \xi_t^2 \tag{1.1}$$

for appropriately chosen positive parameters α_1 and α_2. This is a basic HP filter augmented with the residuals ξ from the following estimated economic relationship.

$$x_t^* = \beta y_t + dX_t + \xi_t,$$

where, x_t^* is another explanatory variable that can be explained by the unobserved variable y, and X is an exogenous variable affected by the parameter d.

There are two different approaches to the question of choosing the smoothing parameters α_1 and α_2. The first approach is to consider the free choice of the smoothing parameters as an advantage. The features of the data can be explored by varying them (see Richardson *et al.* (2000) and Trimbur (2006) for instance). The other view often used by macreconomists suggests that the choice of α_1 and α_2 should be on the basis of priors about the relative variance of each of the terms in (1.1); the higher the variance of the term the lower weight it receives in the minimization process (see Laxton-Tetlow (1992) and Boone (2000)).

The main result of this paper is an optimality criterium for selecting the *best* smoothing parameters and construct consistent estimators of them.

The optimal smoothing parameters for the HPMV filter are pairs of positive parameters (α_1, α_2) which minimize the gap (using the Euclidean norm) between $\hat{y}((\alpha_1, \alpha_2), (x, z))$, that solves (1.1), and $E[\,y|\,x]$,

$$\alpha^* = \arg\min_{(\alpha_1,\alpha_2)} \|E[\,y|\,(x,z)] - \hat{y}((\alpha_1, \alpha_2), (x, z))\|^2, \text{ for all realisations of } (x, z),$$

where, $z = x^* - dX$.

The organisation of this paper is as follows. In Section 2 we describe the HPMV filter, and suggest an optimality criterium for choosing the best smoothing parameters, where we show that there is a whole set of such optimal solutions. We also show that the noise-to-signal ratios used in the macroeconomic literature are optimal as well. In section 3, we apply the recent results obtained in Dermoune *et al.* (2007) to find consistent estimators of these noise-to-signal ratios. We also present a numerical example based on a simulation study that highlights the performance of these estimators.

2 The Hodrick-Prescott Multivariate filter

As a general principle, a macroeconomic time series can be decomposed into its seasonal variation, a 'business-cycle' component , irregular short-term movements, and its long-term trend component. It is standard practice for economic series to be seasonally adjusted. The univariate HP filter is then applied to the seasonally adjusted series to obtaine cyclical residual which is an estimate of the combined cyclical and irregular component of the series.

Let $x = (x_1, ..., x_T) \in \mathbb{R}^T$ be a time series of observables. The HP filter decomposes x into a nonstationary trend $y \in \mathbb{R}^T$ and a cyclical residual component (noise term) $u \in \mathbb{R}^T$:

$$x = y + u. \tag{2.1}$$

Given a smoothing parameter $\alpha > 0$, this decomposition of x is obtained by minimizing the weighted sum of squares

$$\|x - y\|^2 + \alpha\|D^2 y\|^2 \tag{2.2}$$

with respect to y, where, for $a \in \mathbb{R}^T$, $\|a\|^2 = \sum_{i=1}^T a_i^2$. Here, $D^2 y$ is the trend disturbance obtained by acting the second order forward shift operator D^2 on the trend $y = (y_1, y_2, \ldots, y_T)$:

$$D^2 y_t := (y_{t+2} - y_{t+1}) - (y_{t+1} - y_t), \qquad t = 1, 2, \ldots, T-2,$$

or, equivalently,

$$D^2 y_t := 2\left(\frac{y_{t+2} + y_t}{2} - y_{t+1}\right), \qquad t = 1, 2, \ldots, T-2,$$

measuring the deviation between the value of the trend at $t+1$, y_{t+1} and the linear interpolation between y_t and y_{t+2}.

In vector form,

$$Py(t) = D^2 y_t, \qquad t = 1, \ldots, T-2, \tag{2.3}$$

where, the shift operator P is the following $(T-2) \times T$-matrix

$$P := \begin{pmatrix} 1 & -2 & 1 & \ldots & \ldots & 0 \\ 0 & 1 & -2 & 1 & \ldots & 0 \\ & \ldots & \ldots & \ldots & \ldots & \\ 0 & \ldots & \ldots & 1 & -2 & 1 \end{pmatrix}. \tag{2.4}$$

The first term in (2.2) measures a goodness-of-fit by minimizing the the deviation between the trend y_t and the observation x_t and the second term is a measure of the degree-of-smoothness which penalizes decelerations in growth rate of the trend component, by minimizing the the deviation between the trend value y_{t+1} and the linear interpolation between y_t and y_{t+2}.

The Hordrick-Prescott Multivariate filter (HPMV in short) seeks to estimate the unobserved variable y as a solution to the following minimization problem

$$\arg\min_y(||x-y||^2+\alpha_1||Py||^2+\alpha_2||\xi||^2), \tag{2.5}$$

given the following dynamics:

$$\begin{cases} x = y+u, \\ z := x^* - dX = \beta y + \xi, \\ Py = v, \end{cases} \tag{2.6}$$

where, x^* is another explanantory variable and X is an exogenous variable affected by the parameter d. Moreover, the noise term u and the signal terms v and ξ are independent Gaussian vectors with mean zero and respective covariance matrices $\sigma_u^2 I_T$, $\sigma_v^2 I_{T-2}$ and $\sigma_\xi^2 I_T$, where, I_T and I_{T-2} denote the $T\times T$ and $(T-2)\times(T-2)$ identity matrices, respectively:

$$\begin{pmatrix} u \\ \xi \\ v \end{pmatrix} \sim \mathcal{N}(0,\Sigma), \tag{2.7}$$

with covariance matrix

$$\Sigma := \begin{pmatrix} \sigma_u^2 I_T & 0 & 0 \\ 0 & \sigma_\xi^2 I_T & 0 \\ 0 & 0 & \sigma_v^2 I_{T-2} \end{pmatrix}.$$

Since P is of rank $T-2$, the signal $v = Py$ does not determine a unique y but rather a set of solutions

$$y := \{P'(PP')^{-1}v + Z\gamma;\ \gamma \in \mathbb{R}^2\},$$

where, the $T \times 2$-matrix Z satisfies

$$PZ = 0, \qquad Z'Z = I_2, \tag{2.8}$$

In view of (2.6), the time series (x,z) can be represented in terms of (u,v,ξ) as

$$\begin{cases} x = u + P'(PP')^{-1}v + Z\gamma, \\ z = \xi + \beta P'(PP')^{-1}v + \beta Z\gamma, \end{cases} \tag{2.9}$$

for some $\gamma \in \mathbb{R}^2$.

In view of (2.9),

$$\begin{pmatrix} x \\ z \\ y \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} Z \\ \beta Z \\ Z \end{pmatrix}\gamma, \Sigma_{xzy}\right), \tag{2.10}$$

with covariance matrix

$$\Sigma_{xyz} = \begin{pmatrix} \sigma_u^2 I_T + \sigma_v^2 Q & \beta\sigma_v^2 Q & \sigma_v^2 Q \\ \beta\sigma_v^2 Q & \sigma_\xi^2 I_T + \beta\sigma_v^2 Q & \beta\sigma_v^2 Q \\ \sigma_v^2 Q & \beta\sigma_v^2 Q & \sigma_v^2 Q \end{pmatrix},$$

where,

$$Q := P'(PP')^{-1}(PP')^{-1}P$$

is a symmetric matrix that together with P and Z satisfies the following properties.

$$PQZP'(PP')^{-1}P + ZZ' = I_T, \;\; QZ = 0, \;\; Z'Z = I_T, \tag{2.11}$$

and, for any $\lambda > 0$,

$$Q[\lambda I_T + Q]^{-1} = [\lambda I_T + Q]^{-1}Q. \tag{2.12}$$

This yields that

$$Q[\lambda I_T + Q]^{-1}Z = 0. \tag{2.13}$$

Moreover, the following identity holds.

$$(I_T + \lambda P'P)^{-1} = ZZ' + Q[\lambda I_T + Q]^{-1}. \tag{2.14}$$

Since the matrix $\big((1+\alpha_2\beta^2)I_T + \alpha_1 P'P\big)$ is positive definite, the unique solution $\hat{y} := y((\alpha_1, \alpha_2, \beta), (x, z))$ to Problem (2.5) is

$$\hat{y} = a(I_T + bP'P)^{-1}w,$$

or, using (2.14),

$$\hat{y} = aZZ'w + aQ[bI_T + Q]^{-1}, \tag{2.15}$$

where, we have set $a := 1/(1+\alpha_2\beta^2)$, $b := \alpha_1/(1+\alpha_2\beta^2)$ and $w := x + \alpha_2\beta z$.

Equation (2.15) defines the descriptive filter that associates a trend y to the time series (x, z) through w, depending on the smoothing parameters α_1, α_2, the disturbance operator P and the parameter β.

2.1 A criterium to find the best smoothing parameters

Following Schlicht (Schlicht (2006), Theorem 1), a way to estimate the smoothing parameters α_1 and α_2 is to let the optimal solution $\hat{y}$ in (2.15) be the best predictor of any trend y given the time series (x, z), i.e.

$$\hat{y} \approx E[\,y|\,(x, z)]. \tag{2.16}$$

In other words, the conditional expectation $E[\,y|\,(x,z)]$ which is usually not smooth, is 'equal' to the smooth trend $\hat{y}$.

In the macroeconomic literature dealing with the HPMV filter such as Laxton and Tetlow (1992), Richardson *et al.* (2000), Boone (2000) and Chagny and Lemoine (2003), it is suggested that the best smoothing parameters are the noise-to-signal ratios $\alpha_1^e = \sigma_u^2/\sigma_v^2$ and $\alpha_2^e = \sigma_u^2/\sigma_\xi^2$. Applying Schlicht's criterium to this filter we will show in Proposition 2.1 below, that there is a whole family of best smoothing parameters to which belong α_1^e and α_2^e.

Since $\hat{y}$ is a (linear) function of w, adapting Schlichts criterium, the best smoothing parameters of the HPMV filter are those positive parameters α_1^* and α_2^* for which

$$\hat{y} - Z\hat{\gamma} = E[\,y|\,w] - Z\gamma, \quad \text{for all realisations of } (x,z), \tag{2.17}$$

where, $\hat{\gamma}$ is the maximum likelihood estimator of γ which we determine using observations from w:

$$\hat{\gamma} = \arg\min_{\gamma}(w - E(w))'\Sigma_w^{-1}(w - E(w))$$

Since, $E(w) = Z\gamma/a$ and $\Sigma_w = \frac{\sigma_v^2}{a^2}[cI_T + Q]$, we get,

$$\hat{\gamma} = aZ'w. \tag{2.18}$$

In view of (2.10), we have

$$E[\,y|\,w] = Z\gamma + aQ[cI_T + Q]^{-1}(w - Z\gamma), \tag{2.19}$$

where,

$$c := \frac{\sigma_u^2 + \alpha_2^2\beta^2\sigma_\xi^2}{(1+\alpha_2\beta^2)^2\sigma_v^2}.$$

From (2.15) and (2.19) we get

$$\hat{y} - E[\,y|\,w] = Z(aZ'w - \gamma) + aQ\left\{[bI_T + Q]^{-1} - [cI_T + Q]^{-1}\right\}. \tag{2.20}$$

Hence, by Eq. (2.17), the best smoothing parameters are those positive α_1^* and α_2^* for which $b = c$, which yields

$$\alpha_1^* = \frac{\sigma_u^2 + \alpha_2^{*2}\beta^2\sigma_\xi^2}{(1+\alpha_2^*\beta^2)\sigma_v^2},$$

or,

$$\alpha_1^*/\alpha_1^e = 1 + \alpha_2^*\beta^2\left(\alpha_2^*/\alpha_2^e - \alpha_1^*/\alpha_1^e\right),$$

where, $\alpha_1^e = \sigma_u^2/\sigma_v^2$ and $\alpha_2^e = \sigma_u^2/\sigma_\xi^2$.

Furthermore,

$$\hat{y}^* - E[\,y|\,x] = Z(aZ'w - \gamma), \tag{2.21}$$

where, $\hat{y}^* := y((\alpha_1^*, \alpha_2^*), (x,z))$.

We have proved the following proposition

Proposition 2.1 *The best smoothing parameters of the HPMV filter according to Criterium (2.17) are those positive parameters* α_1^* *and* α_2^* *for which*

$$\alpha_1^*/\alpha_1^e = 1 + \alpha_2^*\beta^2\left(\alpha_2^*/\alpha_2^e - \alpha_1^*/\alpha_1^e\right), \tag{2.22}$$

where, $\alpha_1^e = \sigma_u^2/\sigma_v^2$ *and* $\alpha_2^e = \sigma_u^2/\sigma_\xi^2$.

In particular,

(1) $\alpha_1^* = \alpha_1^e$ *if and only if* $\alpha_2^* = \alpha_2^e$.

(2) *If* $\alpha_1^*/\alpha_1^e < 1$, *then necesserily* $\alpha_2^*/\alpha_2^e < \alpha_1^*/\alpha_1^e$.

(3) *If* $\alpha_1^*/\alpha_1^e > 1$, *then necesserily* $\alpha_2^*/\alpha_2^e > \alpha_1^*/\alpha_1^e$.

2.2 Optimality of the best smoothing parameters

In the main result of this section, Theorem 2.2, we will show that the best smoothing parameters α_1^* and α_2^* given in Proposition 2.1 minimize the gap between the descriptive HPMV filter $\hat{y} := y((\alpha_1,\alpha_2),(x,z))$ and the best predictor of the trend y given w, $E[\,y|\,w]$, i.e.

$$(\alpha_1^*,\alpha_2^*) = \arg\min_{\alpha_1>0,\alpha_2>0} \|\hat{y} - E[\,y|\,w]\|,$$

and give an explicit formula for the expected value of the optimal quadratic gap. In particular, it follows that the noise-to-signal ratios α_1^e and α_2^e are optimal.

Theorem 2.2 *We have*

$$(\alpha_1^*,\alpha_2^*) = \arg\min_{\alpha_1>0,\alpha_2>0} \|\hat{y} - E[\,y|\,w]\|, \tag{2.23}$$

Moreover, the optimal gap

$$\hat{y}^* - E[\,y|\,x] = Z(aZ'w - \gamma)$$

is a centered Gaussian vector with covariance matrix

$$cov\left(Z\left(aZ'w-\gamma\right)\right) = \rho ZZ'.$$

where, $\hat{y}^* := y((\alpha_1^*,\alpha_2^*),(x,z))$ *and* $\rho := (\sigma_u^2 + \alpha_2^{*2}\beta^2\sigma_\xi^2)/(1+\alpha_2^*\beta^2)^2$.

In particular,

$$E[\|\hat{y}^* - E[\,y|\,x]\|^2] = E[\|Z\left(\gamma - Z'x\right)\|^2] = \rho\ trace(ZZ').$$

Proof. Recall that, by (2.20), we have

$$\hat{y} - E[\,y|\,w] = Z(aZ'w-\gamma) + aQ\left\{[bI_T+Q]^{-1} - [cI_T+Q]^{-1}\right\}.$$

Now, since $Z'Q = 0$, we also have

$$(Z(aZ'w - \gamma))' \, aQ \left\{ [bI_T + Q]^{-1} - [cI_T + Q]^{-1} \right\} = 0.$$

Hence,

$$\|\hat{y} - E[\, y|\, x]\|^2 = \|Z\,(aZ'w - \gamma)\,\|^2 + \left\| aQ \left\{ [bI_T + Q]^{-1} - [cI_T + Q]^{-1} \right\} x \right\|^2.$$

This yields, using (2.21),

$$\|\hat{y} - E[\, y|\, x]\| \geq \|\hat{y}^* - E[\, y|\, x]\| = \|Z\,(\gamma - Z'x)\,\|,$$

for all positive α_1 and α_2. Thus α_1^* and α_2^* are optimal.

The rest of the proofs is straightforward. □

3 Estimation of the noise-to-signal ratios

In this section we propose consistent estimators of the noise-to-signal ratios $\alpha_1^e = \sigma_u^2/\sigma_v^2$ and $\alpha_2^e = \sigma_u^2/\sigma_\xi^2$, and the parameter β as ratios of explicit unbiased consistent estimators of the variances σ_u^2, σ_v^2 and σ_ξ^2. To this end, we follow Dermoune *et al.* (2007), Section 3.2, to estimate σ_u^2, σ_v^2, using the fact that

$$Px = v + Pu \sim \mathcal{N}(0, \sigma_v^2 I_{T-2} + \sigma_u^2 PP'). \tag{3.1}$$

We get the following unbaised consistant estimates of σ_u^2 and σ_v^2.

$$\widehat{\sigma}_u^2 = -\frac{1}{4(T-3)} \sum_{j=1}^{T-3} Px(j)Px(j+1). \tag{3.2}$$

and

$$\widehat{\sigma}_v^2 = \frac{1}{T-2} \sum_{j=1}^{T-2} Px(j)^2 + \frac{3}{2(T-3)} \sum_{j=1}^{T-3} Px(j)Px(j+1). \tag{3.3}$$

Similarily, to estimate σ_ξ^2, we use the fact that the time series

$$Pz \sim \mathcal{N}(0, \beta^2 \sigma_v^2 + \sigma_\xi^2 PP'). \tag{3.4}$$

is Gaussian and stationary, with the following covariance matrix $(V(i,j))_{ij}$:

$$V(i,j) = \beta^2 \sigma_v^2 \delta_i^j + \sigma_\xi^2 (PP')_{ij} = r_{|i-j|},$$

where,

$$r_k = \begin{cases} \beta^2 \sigma_v^2 + 6\sigma_\xi^2, & \text{if } k = 0; \\ -4\sigma_\xi^2, & \text{if } k = 1; \\ \sigma_\xi^2, & \text{if } k = 2; \\ 0, & \text{otherwise,} \end{cases} \tag{3.5}$$

whose unbiased consistent estimator is explicitly given by

$$\hat{r}_k = \frac{1}{(T-2)-k} \sum_{j=1}^{T-2-k} Pz(j)Pz(j+k), \qquad k = 0, 1, 2.$$

From that we derive the following consistent unbiased estimator of σ_ξ^2

$$\widehat{\sigma}_\xi^2 = -\frac{1}{4}\hat{r}_1 = -\frac{1}{4(T-3)} \sum_{j=1}^{T-3} Pz(j)Pz(j+1). \tag{3.6}$$

Finally, in view of (3.5), combining (3.2), (3.3) and (3.6), we get the following consistent estimators for α_1^e, α_2^e and β.

Theorem 3.1 *The following statistics*

$$\widehat{\alpha}_1^e = -\frac{1}{4}\left(\frac{3}{2} + \frac{(T-3)\sum_{j=1}^{T-2} Px(j)^2}{(T-2)\sum_{j=1}^{T-3} Px(j)Px(j+1)}\right)^{-1}, \tag{3.7}$$

$$\widehat{\alpha}_2^e = \frac{\sum_{j=1}^{T-3} Px(j)Px(j+1)}{\sum_{j=1}^{T-3} Pz(j)Pz(j+1)}, \tag{3.8}$$

and

$$\widehat{\beta} = \left(\frac{2(T-3)\sum_{j=1}^{T-2} Pz(j)^2 + 3(T-2)\sum_{j=1}^{T-3} Pz(j)Pz(j+1)}{2(T-3)\sum_{j=1}^{T-2} Px(j)^2 + 3(T-2)\sum_{j=1}^{T-3} Px(j)Px(j+1)}\right)^{1/2}, \tag{3.9}$$

based on the time series of observations Px and Pz, are consistent estimators of the smoothing parameters α_1^e, α_2^e and the parameter β.

To illustrate the performance of the estimators $\hat{\alpha}_1^e$, $\hat{\alpha}_2^e$ and $\hat{\beta}$, we give a numerical example based on simulations drawn from the distributions of Px and Pz for different values of the time horizon T, and specified values of α_1^e, α_2^e and β.

The estimator $\hat{\alpha}_1^e$ of α_1^e is exactly the one obtained in Dermoune *et al.* (2007) for the standard HP filter, thus suffering from the same deficiency in the sense that it may not be good enough to estimate values of α_1^e larger than 1. Therefore, we restrict our simulation exercise, reported in the numerical example below, to the case $\alpha_1^e \leq 1$.

In Tables 1, 2 and 3, we report on the performance of different estimates of the statistics $\hat{\alpha}_1^e$, $\hat{\alpha}_2^e$ and $\hat{\beta}$, based on 1000 simulations drawn from the distributions of Px and Pz for the values of $T-2$ (the dimension of the matrix PP') ranging from 500 to 5000, and for the following sets of true values of the paramteres; ($\alpha_1^e = 1$, $\alpha_2^e = 1$, $\beta = 0.5$), ($\alpha_1^e = 1$, $\alpha_2^e = 0.5$, $\beta = 2$) and ($\alpha_1^e = 1$, $\alpha_2^e = 16$, $\beta = 0.2$), where, the last set of values is borrowed from Boone (2000).

T-2	500	1000	5000
(mean($\hat{\alpha}_1^e$), std($\hat{\alpha}_1^e$))	(1.13, 0.61)	(1.05, 0.33)	(1.00,0.11)
(mean($\hat{\alpha}_2^e$), std($\hat{\alpha}_2^e$))	(1.01, 0.16)	(1.00, 0.11)	(1.00, 0.05)
(mean($\hat{\beta}$), std($\hat{\beta}$))	(0.47, 0.29)	(0.47, 0.22)	(0.49, 0.08)

Table 1: Performance of different estimates of the statistics on $\alpha_1^e = 1$, $\alpha_2^e = 1$ and $\beta = 0.5$ based on 1000 simulations.

T-2	500	1000	5000
(mean($\hat{\alpha}_1^e$), std($\hat{\alpha}_1^e$))	(1.13, 0.61)	(1.05, 0.33)	(1.00,0.11)
(mean($\hat{\alpha}_2^e$), std($\hat{\alpha}_2^e$))	(0.50, 0.09)	(0.50, 0.06)	(0.50, 0.02)
(mean($\hat{\beta}$), std($\hat{\beta}$))	(2.06, 0.43)	(2.01, 0.23)	(2.00, 0.10)

Table 2: Performance of different estimates of the statistics on $\alpha_1^e = 1$, $\alpha_2^e = 0.5$ and $\beta = 2$ based on 1000 simulations.

T-2	500	1000	5000
(mean($\hat{\alpha}_1^e$), std($\hat{\alpha}_1^e$))	(1.13, 0.61)	(1.05, 0.33)	(1.00,0.11)
(mean($\hat{\alpha}_2^e$), std($\hat{\alpha}_2^e$))	(16.30, 2.73)	(16.14, 1.84)	(15.96, 0.84)
(mean($\hat{\beta}$), std($\hat{\beta}$))	(0.19, 0.05)	(0.20, 0.03)	(0.19, 0.01)

Table 3: Performance of different estimates of the statistics on $\alpha_1^e = 1$, $\alpha_2^e = 16$ and $\beta = 0.2$ based on 1000 simulations.

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