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PUBLICATIONS LIST

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PREPRINTS (4 total):

1. K. Adiprasito and B. Benedetti: *Shellability, Subdivisions, and the Zeeman Conjecture.* Preprint (Feb. 2012, 12 pages) at `arxiv:1202.6606`.

   We prove that the $(d-2)$-nd barycentric subdivision of every convex d-ball is shellable. This yields a new characterization of the PL property in terms of shellability: A sphere or a ball is PL if and only if it becomes shellable after sufficiently many barycentric subdivisions. This improves results by Whitehead, Zeeman and Glaser.

   Moreover, we show the Zeeman conjecture is equivalent to the statement "the product of any contractible 2-complex with an interval becomes (simplicially) collapsible after a suitable number of barycentric subdivisions". This number cannot be bounded: For any two positive integers m, n, we construct a 2-complex C such that $\mathrm{sd}^{\,m}(C \times I^n)$ is not collapsible.

2. K. Adiprasito and B. Benedetti: *Tight complexes in* 3*-space admit perfect discrete Morse functions.* Preprint (Feb. 2012, 12 pages) at `arxiv:1202.3390`.

   In 1967, Chillingworth proved that all convex simplicial 3-balls are collapsible. Using the classical notion of tightness, we generalize this to arbitrary manifolds: We show that all tight simplicial 3-manifolds admit some perfect discrete Morse function. We also strengthen Chillingworth's theorem by proving that all convex simplicial 3-balls are non-evasive. In contrast, we show that many non-evasive 3-balls are not convex.

3. K. Adiprasito and B. Benedetti: *Metric Geometry and collapsibility.* Submitted. Preprint (Jan. 2012, 35 pages) at `arXiv:1107.5789`.

   Cheeger's finiteness theorem bounds the number of diffeomorphism types of manifolds with bounded curvature, diameter and volume; the Hadamard–Cartan theorem, as popularized by Gromov, shows the contractibility of all non-positively curved simply connected metric length spaces. We establish a discrete version of Cheeger's theorem (*"In terms of the number of facets, there are only exponentially many geometric triangulations of Riemannian manifolds with bounded geometry"*), and a discrete version of the Hadamard–Cartan theorem (*"Every complex that is* CAT(0) *with a metric for which all vertex stars are convex, is collapsible"*). The first theorem has applications to discrete quantum gravity; the second shows that Forman's discrete Morse theory may be even sharper than classical Morse theory, in bounding the homology of a manifold. In fact, although Whitehead proved in 1939 that all PL collapsible manifolds are balls, we show that some collapsible manifolds are not balls.

   Further central consequences of our work are:

   (1) Every flag connected complex in which all links are strongly connected, is Hirsch. (This strengthens a result by Provan–Billera.)
   (2) Any linear subdivision of the d-simplex collapses simplicially, after $d-2$ barycentric subdivisions. (This presents progress on an old question by Kirby and Lickorish.)
   (3) There are exponentially many geometric triangulations of S^d. (This interpolates between the result that polytopal d-spheres are exponentially many, and the conjecture that all triangulations of S^d are exponentially many.)
   (4) If a vertex-transitive simplicial complex is CAT(0) with the equilateral flat metric, then it is a simplex. (This connects metric geometry with the evasiveness conjecture.)
   (5) The space of phylogenetic trees is collapsible. (This connects discrete Morse theory to mathematical biology.)

4. B. Benedetti: *Discrete Morse theory is at least as perfect as Morse theory.* Preprint (Jan. 2012, 17 pages) at `arXiv:1010.0548`.

   In bounding the homology of a manifold, Forman's Discrete Morse theory recovers the full precision of classical Morse theory: Given a PL triangulation of a manifold that admits a Morse function with c_i critical points of index i, we show that some subdivision of the triangulation admits a boundary-critical discrete Morse function with c_i interior critical cells of dimension $d-i$. This dualizes and extends a recent result by Gallais. Further consequences of our work are:

(1) Every simply connected smooth d-manifold ($d \neq 4$) admits a locally constructible triangulation. (This solves a problem by Živaljeviuć.)

(2) Up to refining the subdivision, the classical notion of geometric connectivity can be translated combinatorially via the notion of collapse depth.

Published (8 total)

1. B. Benedetti: *Discrete Morse theory for manifolds with boundary.*
   To appear in **Trans. Amer. Math. Soc.** Preprint (33 pages) at `arXiv:1007.3175`.

   We introduce a version of discrete Morse theory specific for manifolds with boundary. The idea is to consider Morse functions for which all boundary cells are critical. We obtain "Relative Morse Inequalities" relating the homology of the manifold to the number of interior critical cells. We also derive a Ball Theorem, in analogy to Forman's Sphere Theorem. The main corollaries of our work are:
   (1) For each $d \geq 3$ and for each $k \geq 0$, there is a PL d-sphere on which any discrete Morse function has more than k critical $(d-1)$-cells.
   (This solves a problem by Chari.)
   (2) For fixed d and k, there are exponentially many combinatorial types of simplicial d-manifolds (counted with respect to the number of facets) that admit discrete Morse functions with at most k critical interior $(d-1)$-cells.
   (This connects discrete Morse theory to enumerative combinatorics/discrete quantum gravity.)
   (3) The barycentric subdivision of any constructible d-ball is collapsible.
   (This "almost" solves a problem by Hachimori.)
   (4) Every constructible ball collapses onto its boundary minus a facet.
   (This improves a result by the author and Ziegler.)
   (5) A 3-ball with a knotted spanning edge cannot collapse onto its boundary minus a facet.
   (This strengthens a classical result by Bing.)

2. B. Benedetti and G. M. Ziegler: *On locally constructible spheres and balls.* **Acta Mathematica** 206 (2011), 205–243.

   Durhuus and Jonsson (1995) introduced the class of "locally constructible" (LC) 3-spheres and showed that there are only exponentially-many combinatorial types of simplicial LC 3-spheres. Such upper bounds are crucial for the convergence of models for 3D quantum gravity.
   We characterize the LC property for d-spheres ("the sphere minus a facet collapses to a $(d-2)$-complex") and for d-balls. In particular, we link it to the classical notions of collapsibility, shellability and constructibility, and obtain hierarchies of such properties for simplicial spheres and balls. The main corollaries of our work are:
   (1) Not all simplicial 3-spheres are locally constructible.
   (This solves a problem by Durhuus and Jonsson.)
   (2) There are only exponentially many shellable simplicial 3-spheres with given number of facets.
   (This answers a question by Kalai.)
   (3) All simplicial constructible 3-balls are collapsible.
   (This answers a question by Hachimori.)
   (4) Not every collapsible 3-ball collapses onto its boundary minus a facet.
   (This property appears in papers by Chillingworth and Lickorish.)

3. B. Benedetti: *Non-evasiveness, collapsibility, and explicit knotted triangulations.*
   Oberwolfach Reports 8, Issue 1 (2011), 403–405.

   We announce an explicit triangulation of a non-collapsible ball with 15 vertices. Moreover, there is a triangulated sphere on 18 vertices on which no discrete Morse function bounds the Betti numbers sharply.

4. B. Benedetti: *Collapses, products and LC manifolds.*
   Journal of Comb. Theory, Series A 118 (2011), 586–590.

   Durhuus and Jonsson (1995) showed that all the LC triangulated 2- and 3-manifolds are spheres. We show here that for each $d \geq 4$ some LC d-manifolds are not spheres. We prove this result by studying how to collapse products of manifolds with exactly one facet removed.

5. B. Benedetti: *Knot theory and robot arms.*
   Oberwolfach Reports 7, Issue 4 (2010), 2732–2735.

   This extended abstract is a brief invitation to knot theory and an exposition of the result by Connelly, Demaine and Rote (2003) on straightening robot arms in the plane.

6. B. Benedetti and F. H. Lutz: *The dunce hat and a minimal non-extendably collapsible 3-ball.*
   To appear in Electronic Geometry Models. Preprint (Dec. 2009, 7 pages) at `arXiv:0912.3723`.

   We present the vertex-minimal triangulation of a 3-ball (and of a 3-sphere) that contains Zeeman's Dunce Hat as subcomplex. While all shellable balls are collapsible, we show that extendably shellable balls need not be extendably collapsible. We also obtain a geometric realization of the 8-vertex Dunce Hat in the Euclidean 3-space.

7. B. Benedetti and M. Varbaro: *Unmixed graphs that are domains.*
   Comm. in Algebra 39 (2011), 2260–2267.

   By characterizing the graphs whose basic covers algebra is an unmixed domain, we extend a theorem by Villareal (2008) from the class of bipartite graphs to the class of all graphs.

8. B. Benedetti, A. Constantinescu and M. Varbaro: *Dimension, depth and* 0*-divisors of the algebra of basic* k*-covers of a graph.* **Le Matematiche**, Vol. 63, No. 2 (2008), 117–156.

   Herzog (2008) introduced the basic covers algebra $A(G)$ of an arbitrary bipartite graph G. We show that both the Krull dimension and the lack of zero-divisors of $A(G)$ admit a purely combinatorial interpretation. This answers a question by Herzog (2008).

Further publications

1. *Gli Archivi della Scienza* (with A. Benedetti). Genoa, Erga (2001), 576 pages.

   A complete guide to all museums, institutions and archives of science and technology in the Italian state. Third volume of the series *Italian Cultural Institutions*, published by Erga.

2. *L'Accademia delle Scienze di Berlino e la sua biblioteca.* Biblioteche Oggi XXVI (2008), 41–47.

   An essay on the history of the Berlin Academy of Sciences and its library, published by an Italian magazine specialized in librarianship.

3. *On Locally Constructible Manifolds.* PhD Thesis.
   Berlin, 2010, 142 pages. Advisor: Günter M. Ziegler, TU Berlin.

   "Locally constructible" (LC) d-manifolds are manifolds obtained from a tree of d-polytopes by repeatedly gluing together two adjacent boundary facets. We characterize them combinatorially as follows: Once deprived of a facet, LC d-manifolds collapse onto the union of their boundary with a $(d-2)$-complex. This reveals unexpected connections with several fields of research, from knot theory to algebraic topology, from commutative algebra to discrete quantum gravity.