Exercise session 8

Problem 1

Let a_j, $j = 1, \ldots, n$ and b be given positive constants. Solve the following problem using Lagrangian relaxation. Motivate global optimality.

$$\begin{aligned}
\text{(P)} \qquad & \min \sum_{j=1}^{n} x_j^2 \\
& s.t. \sum_{j=1}^{n} a_j x_j \geq b \\
& \quad x_j \geq 0, \qquad j = 1, \ldots, n.
\end{aligned}$$

Problem 2

Consider the following problem (P) in the variables x_1, x_2 and x_3

$$\begin{aligned}
\text{(P)} \qquad & \min \frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + 9x_1 + 16x_2 + 25x_3 \\
& s.t. \frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} \leq b \\
& \quad x_j > 0, \qquad j = 1, 2, 3.
\end{aligned}$$

where $b > 0$ is a given strictly positive number.
Let the constraints $\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} - b \leq 0$ be the only explicit constraint ($g(\mathbf{x}) \leq 0$), while the constraints $x_j > 0$ are considered to be implicit constraints ($\mathbf{x} \in X$).

a) Use Lagrange relaxation (with respect to the explicit constraint) to deduce an *explicit* expression for the dual objective function $\varphi(y)$, valid for $y \geq 0$.

b) Assume that b = 6.
Calculate a number $\hat{y} \geq 0$ such that $\varphi(\hat{y}) \geq \varphi(y)$ for all $y \geq 0$.
Then calculate a vector $\hat{\mathbf{x}} \in X$ which together with $\hat{y}$ satisfies the global optimality condition (GOC) for (P).
Finally, check that the primal objective value for $\mathbf{x} = \hat{\mathbf{x}}$ is equal to the dual objective value for $y = \hat{y}$.

c) Repeat all the steps in the above exercise (b), but now with b=18.

Note that this is Problem 4 on SF1811 Exam 2017-01-11.