First Predictions of the "Anyon": History and Some Wider Perspectives

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Overview of my talk

1. "Anyons" and "nonabelian anyons in two-space": The easiest of ideas
2. Why so long? Profound epistemological and cognitive obstacles
3. Antecedent ideas: Aharonov-Bohm effect, Feynman paths, topology in quantum mechanics
4. Further antecedent ideas: Group representations, induced representations, local current algebras, infinite-dimensional groups; parastatistics; quantum field theory, braid group; 2D statistical mechanics
5. Three independent predictions
6. Implications for educating future mathematicians and physicists: creative activity and learning
7. Issues of acknowledgment, and their wider consequences: academic integrity, risk-taking, fairness and equity

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Note: There followed an extensive literature on now well-known theoretical applications of anyon and non-abelian anyon statistics – condensed matter physics, quantum Hall effect, quantum vortices, quantum field theory, quantum computing, etc. – which is beyond the scope of this discussion.

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1. “Anyons” and “nonabelian anyons in two-space”: The easiest of ideas

The basic idea behind “anyons” is easy.

Let two indistinguishable particles in the plane exchange positions (without passing “through” each other). Then we can consistently keep track of the number of (clockwise or counterclockwise) windings of one about the other.

This is not true in three or more space dimensions.

So instead of limiting ourselves to a factor of +1 or -1, the wave function could pick up a phase $\exp i\theta$.

Two clockwise exchanges in succession are inequivalent to making no exchange. So we need not set $\exp 2i\theta = 1$.

When indistinguishable particles exchange position, predictions of observables should be invariant. This will still hold.

So why did this take half a century – from understanding bosons and fermions, to predicting intermediate statistics?

What then led to three independent predictions, in close succession?

2. Why so long? Profound epistemological and cognitive obstacles

The concept of “epistemological obstacle”

Historical paths of discovery, conceptual schemes, and the meaning of representations

Examples from mathematics

Examples from physics; dependence on the accessibility of empirical observations

The concept of “cognitive obstacle”:

Psychology of individual thinkers; belief structures

"Ontogeny recapitulates phylogeny"

Analogy with learning pathways: individuals’ stages of learning (breaking through cognitive obstacles) recapitulate historical processes (overcoming epistemological obstacles)

Obstacles to “anyons”

The meaning of particle exchange for indistinguishable particles

Labels and the exchange of labels

Index permutations vs. value permutations

Configuration space and coordinate space

Empirical knowledge of fundamental particles: only two kinds (bosons and fermions), falling within powerful, all-embracing theories (Pauli exclusion principle, Bose-Einstein condensation, etc.)

Single-valuedness of wave functions

Bohr atom and quantization

Schrödinger equation: wave functions continuous on a physical space, satisfying boundary conditions

Axiomatic approach to quantum mechanics: unintended consequences

“Stronger than necessary” axioms: Wanting to prove well-established beliefs (totally symmetric or totally antisymmetric wave functions) as theorems, we write axioms to facilitate this.

3. Antecedent ideas

These are the developments that enabled the overcoming of epistemological (and cognitive) obstacles:

Aharonov-Bohm effect:

Charged particles circling regions of magnetic flux

Role of topology in quantum mechanics

Multivalued wave functions achieve legitimacy

Possible physical meaning of the electromagnetic potential A_μ (open to question)

Alternative self-adjoint extensions of differential operators

Feynman path integral formulation and topology

Paths from initial to final state: particle locations traverse paths (tacit implication: indices do not)

Feynman path model for spin, statistics: Lawrence Schulman; Michael Laidlaw and Cécile Morette DeWitt; J. S. Dowker

4. Further antecedent ideas: Group representations, parastatistics, braid group, etc.

Group representations

Unitary representations of Lie groups fundamental to quantum mechanics; theory of induced representations: Eugene Wigner, George Mackey (quarks and high-energy particle physics: Murray Gell-Mann; Yuval Ne'eman; George Zweig)

Current algebra and infinite-dimensional groups: Roger Dashen, Stephen Adler

Nonrelativistic current algebra and diffeomorphism group representations: Dashen, David Sharp, GG, Ralph Menikoff

Rotation generators in two-space: modeling the A-B effect with a covering group: Christiane Martin

Further antecedent ideas (continued)

Parastatistics

Quantum fields (trilinear brackets) and higher dimensional representations of S_N: Herbert S. Green; Albert Messiah and Oscar Wallace Greenberg;

Opens consideration of exotic statistics

Possible application to quarks (superseded by description via additional quantum numbers)

Quantum field theory, braid group

Analogous concept in quantum field-theoretic models with soliton fields in two dimensions: Ray Streater and Ivan Wilde; Jürg Fröhlich

5. Three independent predictions

Leinaas and Myrheim (1977)

Homotopy classes Feynman paths, singular points where particles coincide

Why can't dynamical paths cross? Possible introduction of hardcore potential

Connection with electromagnetism

Goldin, Menikoff, and Sharp (1980-81)

Mathematically rigorous prediction from classifying unitary representations of the diffeomorphism group of $\mathbf{R}^2$

Wave functions single-valued on configuration space (exchange statistics encoded in operators describing local currents); equivariant wave functions defined on the universal covering space

Shift in kinetic angular momentum and energy spectra

(continued)

Three independent predictions (continued)

Goldin, Menikoff, and Sharp (continued)

No arbitrariness in excluding coincidence points; no hard-core potential required

First (to my knowledge) identification of the braid group governing N-anyon statistics (1983); first (to my knowledge) prediction of “nonabelian anyons” equivariant under higher-dimensional unitary braid group representations (1985)

Wilczek (1982)

Investigation of fractional quantum numbers leads to fractional spin in two dimensions

Model based on charged particles bound to units of magnetic flux (“particle-flux tube composites”)

Coinage of the name “anyons” (which caught on) for particles satisfying “any” statistics interpolating bosons and fermions

Suggestion of a fractional spin-statistics connection

Subsequent powerful advocacy for the theoretical importance of anyons

6. Educational implications

What can we learn from this history of ideas about educating future (or current) mathematicians and physicists? Or educating ourselves?

What does it mean for a student to "really understand" a newly-studied concept in mathematics or physics? What *should* it mean?

How to state and apply the concept ... presently emphasized

Knowing the accepted rationale for the concept ... usually explained

Understanding the "why" behind the concept ... sometimes developed

Questioning the foundations of the concept ... less frequently encouraged

Seeing how the concept might have been invented for the first time, by someone who didn't know it ... rarely explored

Thinking through how the student herself or himself could have invented the concept, without having ever seen it ... almost never considered

Implications for education (continued)

Encouraging creativity in mathematics and physics

In art and literature: Creativity includes breaking conventions, introducing new forms or novel, exciting variations. It is relatively unconstrained.

In architecture: New forms are subject to constraints of functionality, safety, etc.

In mathematics: Israel Gelfand sugests mathematics shares four characteristics with classical music and poetry:* (1) Beauty, (2) Symplicity, (3) Exactness, and most importantly (4) Crazy ideas.

*Talk by I.M. Gelfand at the Royal East Restaurant, Cambridge, Mass., Sept. 2003.

In physics, "crazy ideas" deserve exploration. Sometimes they take courage to express. The constraints are ultimately empirical as well as logical – but if an idea is logical, it can even enable an empirical bound on its occurrence. The history of anyons includes many ideas that seemed "crazy" at the time.

In math and science education: Emphasize exploration and discovery from an early age. Promote open-ended problem solving, and experiments without predetermined outcomes. Hold high aspirations for all, girls as well as boys, children of all races and nations.

7. Issues of truthful acknowledgment, and their wider consequences

Introduction

This is difficult subject to discuss, because it's very personal.

At the same time, it's important for the field, and for many others who are less able to raise the issue securely.

It is also part of the history of anyons, and has affected it.

Some background, and current events: No dispute – but after 40 years, no acknowledgment:

Experimental confirmation of anyonic excitations (2020)

Discover magazine (2020, 2021)

Quanta magazine (2021)

Wikipedia, "anyon" entry (current)

Issues of acknowledgment (continued)

Acknowledging the physicists who first predicted “anyons”:

1980-89

Early failures of acknowledgment, some inadvertent but some deliberate, have long-enduring effects.

Correspondence by letter then was slow, difficult, and essentially ineffective

Risk-taking and reward: Power relationships influence acknowledgment, with psychological and career consequences.

The meaning for scientific investigation during this period: some methods received attention, others were disregarded. Missed opportunities.

Issues of acknowledgment (continued)

1989-1991

Physics Today (1989, 1990) – A battle and its favorable resolution, but ...

Scientific American (1991) and elsewhere – the struggle continues, though the truth is undisputed.

Recent events (2020-21)

Experimental confirmation of anyonic excitations – and the systemic problem persists ...

Discover magazine – Intentionally dishonest journalism in science.

Quanta magazine – Mere disregard of communication.

Wikipedia ("anyon" entry) – An anonymous editor (screen name "HouseofChange") systematically removes correct citations, and levels hostile and false accusations.

Why does it matter? A wider perspective

Acknowledgment is a personal issue, yes, but it is one affecting the whole community of science. Many, many others have had similar experiences.

Academic integrity in science: Are we respecting truth, and valuing integrity? Are we rewarding science, or fame and promotional capability instead?

Risk-taking: Original ideas, and predictions of previously unknown or unanticipated phenomena, involve risk – the investment of time, the possible loss of career opportunities, even reputational risk. Success is far from assured. When even dramatic success is unrecognized, disillusionment enters.

Fairness and equity: The absence of truthful acknowledgment in the sciences is a factor that differentially affects women, Black people, some graduate students and post-docs, researchers from developing countries, and those without "connections".

It may one day affect some of you, or even be affecting your opportunities now.

Conclusion

The loss when correct citation is withheld is not only to individuals. It is a denial of information, detrimental to the progress of science and its benefits to humanity.

David Sharp and I are extremely fortunate. We are among those who have had great opportunities: fellowships, fine educations at the best universities, successful careers, honors and awards in physics. We took risks in our early careers, pursued less-traveled paths, endured some difficult consequences, and emerged as continuing, active scientists.

Unfortunately, it was only our positions, status, and connections that enabled us to achieve the earlier corrections published in *Physics Today* and *Scientific American*.

Today, those without such resources have little or no recourse – and we, too, encounter impasse after impasse.

We both feel it is our obligation to the field to speak up. The outcome will not affect us materially, but the process – however difficult – is necessary toward solving a serious and systemic problem.

Some additional references, for those interested in our approach to anyons

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Thank you for your attention.